# NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Contract NGR 14-008-002

# STRUCTURAL DESIGN CONCEPTS FOR FUTURE SPACE MISSIONS

Progress Report for period
May 1, 1968 to October 31, 1968
FACULTY FORM 602

N69-29417
(ACCESSION NUMBER)
63 PAGES
CODE 1
CR#101577
(NASSA CR OR TMX OR AD NUMBER)
CATAGORY 19

**THE SCHOOL OF TECHNOLOGY SOUTHERN ILLINOIS UNIVERSITY**

STRUCTURAL DESIGN
CONCEPTS FOR FUTURE
SPACE MISSIONS

National Aeronautics and
Space Administration
November 1, 1968

Progress Report
NASA Contract
NGR 14-008-002

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## NOTICES

Author's note:

All images and text are adapted from original NASA document, recreated or made from scratch by Jay Salsburg. wizard@prysm.net

## FOREWARD

This report was prepared in the School of Technology at Southern Illinois University under National Aeronautics and Space Administration Contract No. NGR 14-008-002.

This report covers the progress of work from May 1, 1968 to October 31, 1968.

Personnel participating in the research include Julian H. Lauchner, principal investigator, R. Buckminster Fuller, Joseph D. Clinton, Mark B. Mabee, Richard M. Moeller, and Richard Flood.

## ABSTRACT

This report explains one method of subdividing a polyhedron into triangular facets and "exploding" it into the surface of a sphere. A mathematical model is included which explains the geometry used in subdividing and transforming the icosahedron into the structural sphere. Also included are a computer program and a plot routine used in the computations.

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## INTRODUCTION

This report explains one method of subdividing a polyhedron into triangular facets and ''exploding" it onto the surface of a sphere. A structure is thereby given which may be used in spherical form.

The tetrahedron, octahedron or icosahedron, are the fundamental geometrical configurations of the structure. A further subdivision of the configuration chosen is accomplished by subdividing each principle side of each principle polyhedral triangle into any number of segments. The order of subdivision is determined by subdividing the triangle and the origin (or center) of the polyhedron into equal angle segments, using the origin as the vertice for subdivision. The points of intersection of the equal angle segments with the principle side determines the subdivision along the principle side of the principle polybedral triangle.

The points of subdivision on each side of the Principle polyhedral triangle are connected with line segments which are parallel to the two remaining sides of the principle polyhedral triangle under consideration. They intersect at a number of points which define a triangular grid of subdivision. Due to the method of subdivision, small triangular "windows" occur in the grid. The centers of these windows are found by one of two methods (discussed later in this report) and are used as the vertices of a triangular grid of subdivision of the principle polyhedral face and are then transformed to the surface of the sphere which circumscribes the polyhedron. The cords that connect these transformed vertices thus define the structural grid that comprise the structural configuration desired.

A mathematical model has been determined which explains the geometry used in subdividing and transforming the polyhedron into the structural configuration desired. From this model a computer program has been written which gives the necessary information needed for construction and analysis of the structure. As a further aid in investigation of the various forms, a plot routine was developed to give a graphical output of each of the structural forms.

## BASIC GEOMETRY

DEFINITIONS:

DIHEDRAL ANGLE ( $\beta$)an angle formed by two planes meeting in a common line. The planes themselves are the faces of the dihedral angle, and the common line is the element. To measure the dihedral **angle measure** the angle whose vertex is on the element of the dihedral angle and whose sides are perpendicular to the element and lie one in each face of the dihedral angle.

FACE ANGLE ($\alpha$) an angle formed by two elements meeting in a common point and lying in a plane that is one of the faces of the polyhedron.

AXIAL Angle ($\Omega$) an angle formed by an element and a radius from the center of the polyhedron meeting in a common point and the vertex of the axial angle sharing a vertex of the polyhedron.

PRINCIPLE ICOSAHEDRAL TRIANGLE **(PIT)** any one of the 20 equal equilateral triangles which forms the face of the regular icosahedron.

PRINCIPLE SIDE any one of the three sides of the principle icosahedral triangle.

FREQUENCY the number of equal parts into which a principle side is subdivided.

FACES the triangles making up the "exploded" structural form.

## GEOMETRY

This program works with a tetrahedron, octahedron, or icosahedron circumscribed by a unit sphere. The icosahedron was chosen as an example to illustrate the geometry of the program. The icosahedron is oriented in a three dimensional rectangular coordinate system so that the vertices of one principle icosahedral triangle are:

$$(\mathrm{x},\ \mathrm{y},\ \mathrm{z}) = \left(0, \frac{\sqrt{\tau}}{\sqrt[4]{5}}, \frac{1}{\sqrt[4]{5}\sqrt{\tau}}\right)$$

=(0, .850651, .525731)

$$(\mathrm{x},\ \mathrm{y},\ \mathrm{z}) = \left(\frac{1}{\sqrt[4]{5}\sqrt{\tau}}, 0, \frac{\sqrt{\tau}}{\sqrt[4]{5}}\right)$$

=(.525731, 0, .850651)

$$(\mathrm{x},\ \mathrm{y},\ \mathrm{z}) = \left(\frac{\sqrt{\tau}}{\sqrt[4]{5}}, \frac{1}{\sqrt[4]{5}\sqrt{\tau}}, 0\right)$$

=(.850651, .525731, 0)

$$\text{where } \tau = \left(\frac{1+\sqrt{5}}{2}\right)$$

with the intersections of the axis X, Y, Z, located at the origin (O,O,O) of the icosahedron, Figure 1.

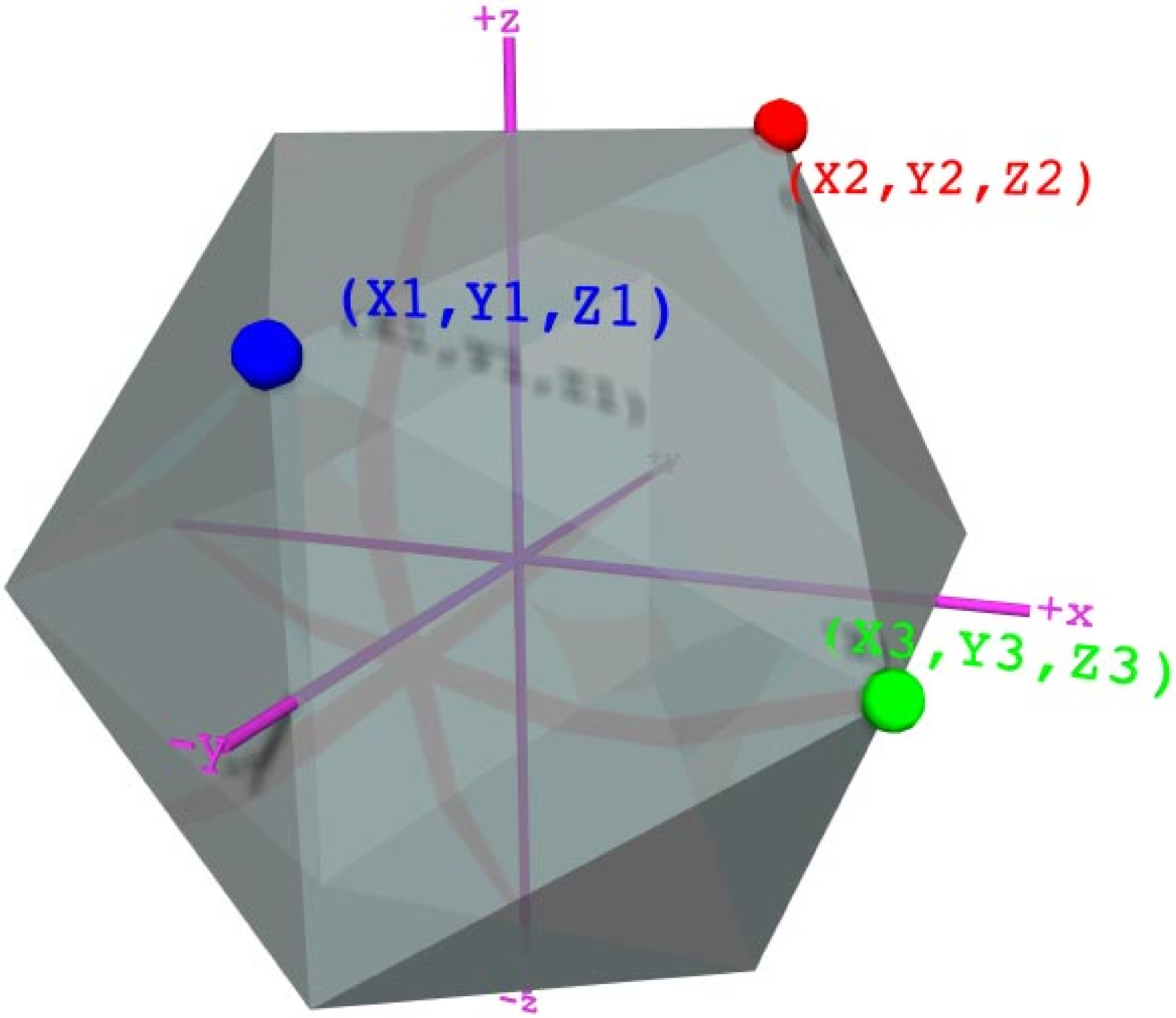


Figure 1

This principle icosahedral triangle is divided into smaller triangular units which are ‘exploded" onto the surface of a sphere constituting the desirable space form.

Using the following formula the planes consisting of the edges of the PIT and the origin

(X1, Y1, Z1) (X2, Y2, Z2) (X3, Y3, Z3)

are rotated from 3-space into 2-space, Figure 2,

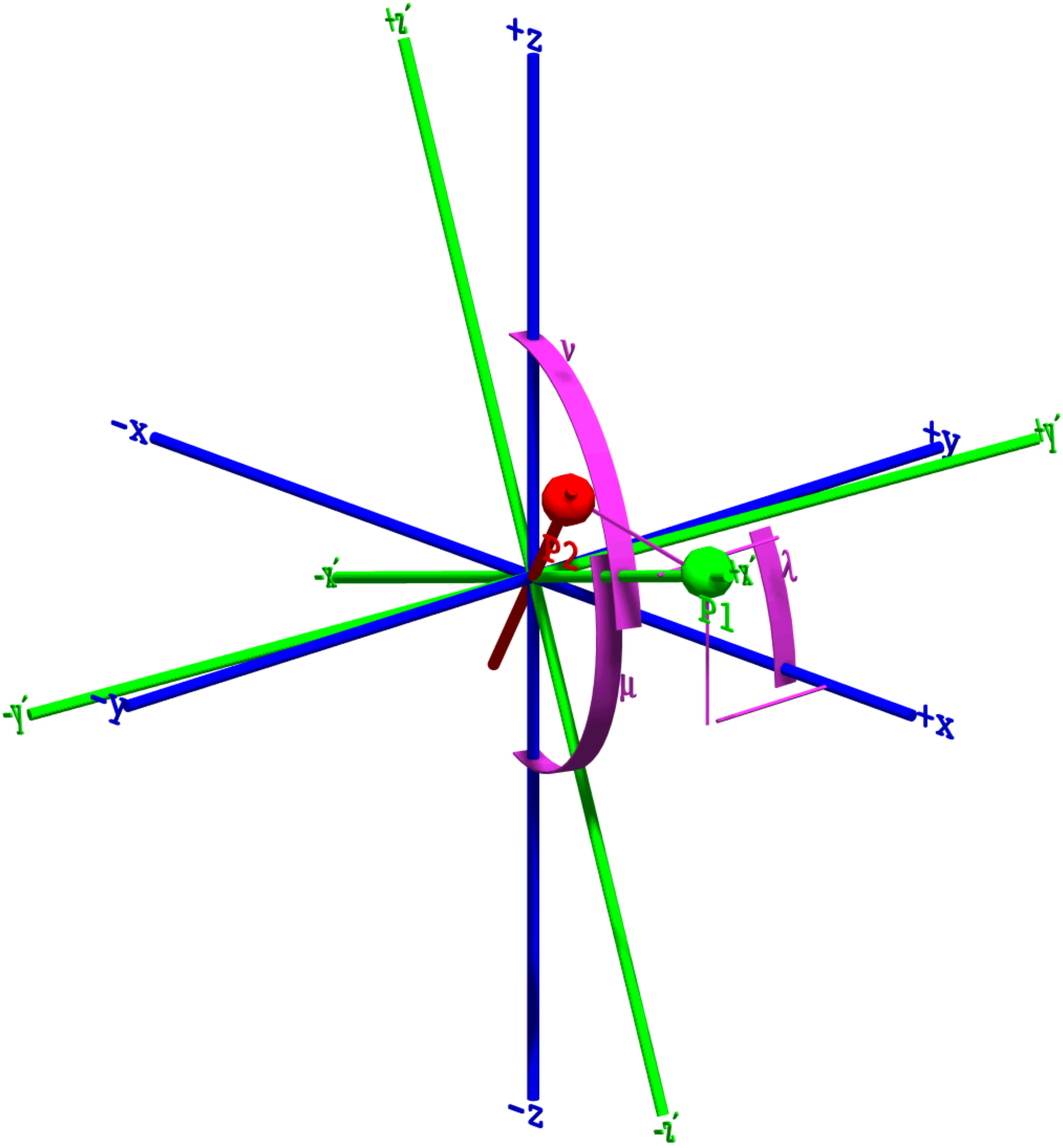


Figure 2.

$$x' = \lambda_1 x + \mu_1 y + \nu_1 z$$
$$y' = \lambda_2 x + \mu_2 y + \nu_2 z$$
$$z' = \lambda_3 x + \mu_3 y + \nu_3 z \qquad \textbf{[1]}$$

Where $\lambda, \mu, \nu$ are direction cosines of the X'-axis, Y'-axis, and Z'-axis respectively with respect to the old axis and are found by:

$$\lambda_1 = x_1 \Big/ \sqrt{x_1^{\ 2} + y_1^{\ 2} + z_1^{\ 2}}$$
$$\mu_1 = y_1 \Big/ \sqrt{x_1^{\ 2} + y_1^{\ 2} + z_1^{\ 2}}$$
$$\nu_1 = z_1 \Big/ \sqrt{x_1^{\ 2} + y_1^{\ 2} + z_1^{\ 2}}$$

$\lambda_2, \lambda_3, \mu_2, \mu_3;$ and $\nu_2, \nu_3$ are found similarly.

The edge of the PIT is subdivided into units by the following method, Figures 3 and 4.

FIND: the angle $\phi$ contained within the rotated triangle consisting of $\overline{P_1P_2}$ . and the origin with the vertex located at the origin.

$$\phi = \text{Arctan}\left(\frac{P_{y_2}}{P_{x_2}}\right) * r \qquad \textbf{[2]}$$

where r = 1 and is considered constant.

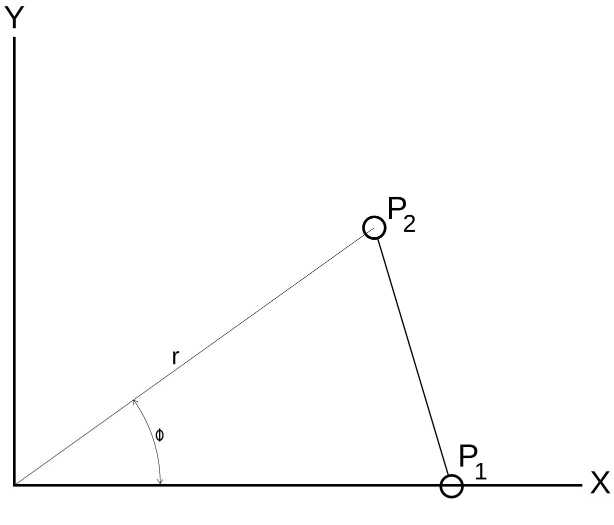


Figure 3.

THEN: subdivide $\phi$ angle into N angles $\theta$

$$\theta = \left(\frac{\phi}{N}\right) * T \qquad \textbf{[3]}$$

where T = Increment 1 to N

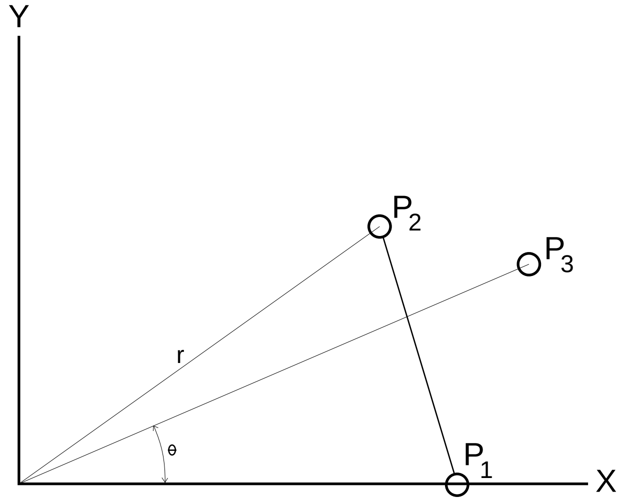


Figure 4.

The points of interest of $\overline{OP_3}$ and $\overline{P_1P_2}$ are found:

$$\overline{P_1P_2} \text{ is } \frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\overline{OP_3} \text{ is } \frac{y - 0}{x - 0} = \frac{y_3 - 0}{x_3 - 0} \qquad \textbf{[4]}$$

The equation takes the following form:

$$\overline{P_1P_2} \text{ is } x(y_2 - y_1) + y(x_1 - x_2) = y_1(x_1 - x_2) + x_1(y_2 - y_1)$$

let

$$\begin{aligned} (y_2 - y_1) &= a_1 \\ (x_2 - x_1) &= b_1 \\ y_1(x_1 - x_2) + x_1(y_2 - y_1) &= C_1 \end{aligned}$$

$$\overline{OP} \text{ is } xy_3 - yx_3 = 0$$

let

$$\begin{aligned} y_3 &= a_3 \\ -x_3 &= b_2 \\ 0 &= c_2 \end{aligned}$$

Solve the equations for the point of intersection:

$$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \quad X = \frac{\begin{vmatrix} c_1 & b_1 \\ c_1 & b_2 \end{vmatrix}}{D} \quad Y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_1 & c_2 \end{vmatrix}}{D} \qquad \textbf{[5]}$$

Rotate the points of intersection along the PIT edge from 2-spaces back to 3-spaces.

$$x = \lambda_1 x' + \mu_1 y' + \nu_1 z$$
$$y = \lambda_2 x' + \mu_2 y' + \nu_2 z$$
$$z = \lambda_3 x' + \mu_3 y' + \nu_3 z$$

where $\lambda, \mu, \nu,$ are direction cosines of the X'-axis, Y'-axis, and Z'-axis with respect to the old axis and are found:

$$\lambda_1 = x_1 \Big/ \sqrt{x_1^2 + y_1^2 + z_1^2}$$
$$\mu_1 = y_1 \Big/ \sqrt{x_1^2 + y_1^2 + z_1^2}$$
$$\nu_1 = z_1 \Big/ \sqrt{x_1^2 + y_1^2 + z_1^2}$$

an $\lambda_2, \lambda_3, \mu_2, \mu_3;$ d $\nu_2, \nu_3$ are found similarly.

Retain the co-ordinates along the edges an $S_1, S_2,$ and $S_3$ as shown in Figure 5.

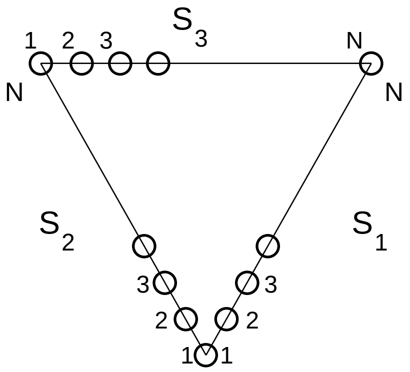


Figure 5

After finding the unit measurements along the edges of the PIT, they are connected through a 3-way grid determining a smaller triangular grid network. Since the units along the PIT edge are not equal, the 3-way gridding will create "windows". The centers of these "windows" must be found to establish the final 3-way grid network, Figure 6.

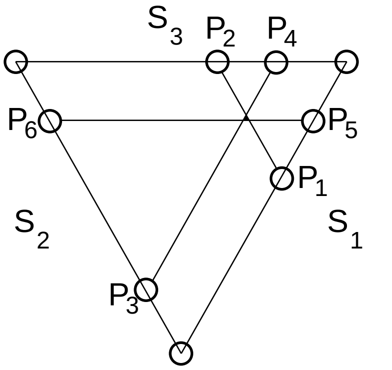


Figure 6.

The gridding and windows are found by the following method: from the coordinates along the edges of S‘,S2 and S3, calculate coordinates of the window by finding the intersection of $\overline{P_1P_2}$ with $\overline{P_3P_4}$ and $\overline{P_1P_2}$ with $\overline{P_5P_6}$ and $\overline{P_3P_4}$ with $\overline{P_5P_6}$ by using the two point form of the equation of a line in three-space for the three lines and solve simultaneously for the points of intersection.

$$\begin{aligned}
\overline{P_1P_2} &= \frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_2} = \frac{z - z_1}{z_2 - z_1} \\
\overline{P_3P_4} &= \frac{x - x_3}{x_4 - x_3} = \frac{y - y_3}{y_4 - y_3} = \frac{z - z_3}{z_4 - z_3} \\
\overline{P_5P_6} &= \frac{x - x_5}{x_6 - x_5} = \frac{y - y_5}{y_6 - y_5} = \frac{z - z_5}{z_6 - z_5}
\end{aligned}$$

**[7]**

To find the intersection of $\overline{P_1P_2}$ with $\overline{P_3P_4}$ the equation takes the following form:

$$\begin{aligned}
&(1)\ \overline{P_1P_2}\text{ is: } x(y_2 - y_1) + y(x_1 - x_2) = y_1(x_1 - x_2) + x_1(y_2 - y_1) \\
&(2)\ \overline{P_1P_2}\text{ is: } y(z_2 - z_1) + z(y_2 - y_1) = z_1(y_1 - y_2) + y_1(z_2 - z_1) \\
&(3)\ \overline{P_3P_4}\text{ is: } x(y_4 - y_3) + y(x_3 - x_4) = y_1(x_3 - x_4) + x_1(y_4 - y_3) \\
&(4)\ \overline{P_3P_4}\text{ is: } y(z_4 - z_3) + z(y_3 - y_4) = z_3(y_3 - y_4) + y_3(z_4 - z_3)
\end{aligned}$$

$$\begin{aligned}
\text{For } \overline{P_1P_2}\text{ let: } \quad (y_2 - y_1) &= a_1 \\
(x_1 - x_2) &= b_1 \\
y_1(x_1 - x_2) + x_1(y_2 - y_1) &= c_1
\end{aligned}$$

$$\begin{aligned}
\text{For } \overline{P_3P_4}\text{ let: } \quad (y_4 - y_3) &= a_2 \\
(x_3 - x_4) &= b_2 \\
y_1(x_3 - x_4) + x_1(y_4 - y_3) &= c_2
\end{aligned}$$

using the formula **[6]** solve for x and y coordinates of the intersections of $\overline{P_1P_2}$ with $\overline{P_3P_4}$.

Find the z coordinate:

$$\begin{aligned}
\text{For } \overline{P_1P_2}\text{ let: } \quad (z_2 - z_1) &= a_1 \\
(y_1 - y_2) &= b_1 \\
z_1(y_1 - y_2) + y_1(z_4 - z_3) &= c_1
\end{aligned}$$

$$\begin{aligned}
\text{For } \overline{P_3P_4}\text{ let: } \quad (z_4 - z_3) &= a_2 \\
(y_3 - y_4) &= b_2 \\
z_1(y_3 - y_4) + y_1(z_4 - z_3) &= c_2
\end{aligned}$$

The other two vertices of the window are found in a similar manner. Once the coordinates for the vertices of the window are determined, its center is found by one of the following two methods:

METHOD I:

On the Principle Icosahedral Triangular Plane the windows appear as equalateral triangles with vertices $P_1(x_1\ y_1\ z_1),\ P_2(x_2\ y_2\ z_2),\ P_3(x_3\ y_3\ z_3)$ as shown in Figure 7.

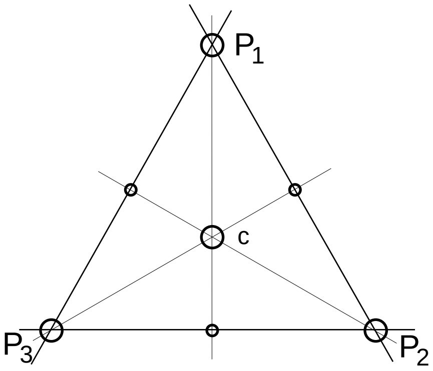


Figure 7.

The center **C(cx,cy,cz)** is found with the following formula:

$$CX = \frac{x_1 + x_2 + x_3}{3}$$
$$CY = \frac{y_1 + y_2 + y_3}{3} \qquad \textbf{[8]}$$
$$CZ = \frac{z_1 + z_2 + z_3}{3}$$

METHOD II:

The coordinates of the window found on the surface of the PIT are first “exploded” to the surface of the sphere. The center of the exploded window is then found by the intersection of angle bisectors. To find the projection of each vertex of the window onto the unit sphere. translate each vertex along a line through the vertex of the PIT and the origin; each coordinate of each vertex, PIT, is divided by the distance between the vertex PIT and the origin, Figure 8.

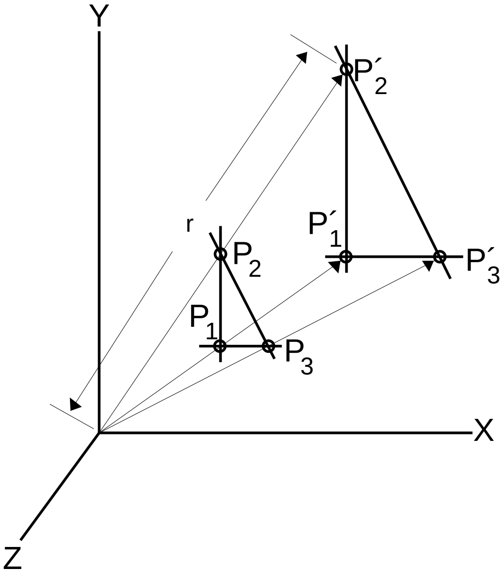


Figure 8.

$$d = \sqrt{x_1^{\,2} + y_1^{\,2} + z_1^{\,2}} \qquad [9]$$

where d = distance from origin to $P_1$

r = 1

where **r** = the radius of the sphere to be exploded upon and is considered constant

$$x'_1 = \frac{rx_1}{d}$$
$$y'_1 = \frac{ry_1}{d} \qquad [10]$$
$$z'_1 = \frac{rz_1}{d}$$

Translate “window’ with vertice P3 at the origin. Figure 9.

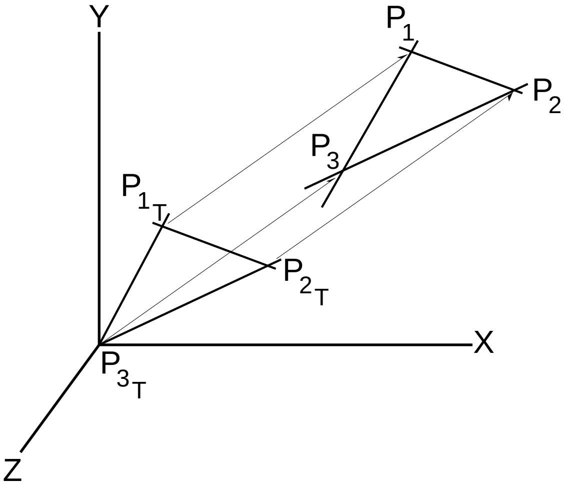


Figure 9.

$$
\begin{aligned}
P_{1Tx} &= P_{1x} - P_{3x}\\
P_{1Ty} &= P_{1y} - P_{3y}\\
P_{1Tz} &= P_{1z} - P_{3z}\\
\\
P_{2Tx} &= P_{2x} - P_{3x}\\
P_{2Ty} &= P_{2y} - P_{3y}\\
P_{2Tz} &= P_{2z} - P_{3z}\\
P_{3Tx_1}, P_{3Ty_1}, P_{3Tz_1} &= 0
\end{aligned}
$$

**[11]**

Rotate plane $P_1, P_2, P_3$ so that $\overline{P_1P_2}$ will fall on the X-axis and $P_3$ is at the origin using equation **[1]**.

The center is found with the intersection of two angle bisectors of triangular window $P_1\ P_2\ P_3$, Figure 10.

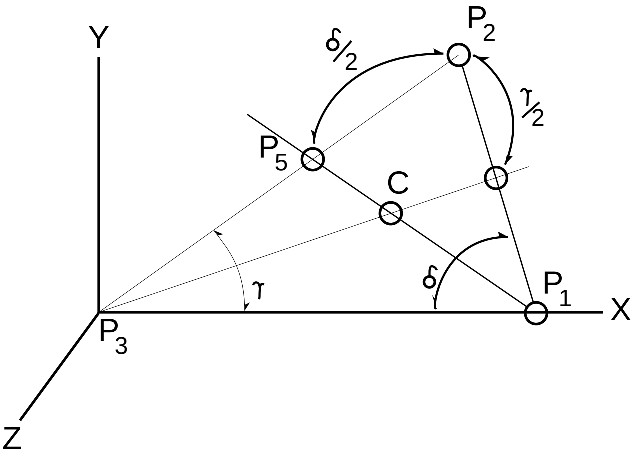


Figure 10.

The angles $\gamma$ and $\delta$ are found:

$$
\begin{aligned}
\text{Arctan}\ \frac{y_2}{x_2} &= \gamma\\
\text{Arctan}\ \frac{y_2}{x_1 - x_2} &= \delta
\end{aligned}
$$

**[12]**

rotate $P_2$ about $P_3$ toward $P_1$, $1/2\ \gamma$ degrees

$$
\begin{aligned}
x_4 &= x_2 \cos 1/2\ \gamma + y_2 \sin 1/2\ \gamma\\
y_4 &= y_2 \cos 1/2\ \gamma - x_2 \sin 1/2\ \gamma
\end{aligned}
$$

**[13]**

locate the origin at $P_1$, then rotate $P_2$ about $P_1$ toward $P_3$ $1/2\ \delta$ degrees.

$$
\begin{aligned}
x_5 &= \left(x_2 - x_1\right) \cos 1/2\ \delta - y_2 \sin 1/2\ \delta + x_1\\
y_5 &= y_2 \cos 1/2\ \delta + \left(x_2 - x_1\right) \sin 1/2\ \delta
\end{aligned}
$$

**[14]**

thus defining $\overline{P_1P_5}$ and $\overline{P_3P_4}$.

With $P_3$ at the origin formula **[4]** may be used to solve for the intersection of line $\overline{P_3P_4}$, $\overline{P_1P_5}$ finding center **C**. Rotate **C** back to three space using formula **[6]**. Then translate center **C** back to three space ("**C**" is located in the previously "exploded" window), Figure 11.

$$
\begin{aligned}
C'x &= Cx \ \ P_3x \\
C'y &= Cy \ \ P_3y \\
C'z &= Cz \ \ P_3z
\end{aligned}
\qquad \textbf{[15]}
$$

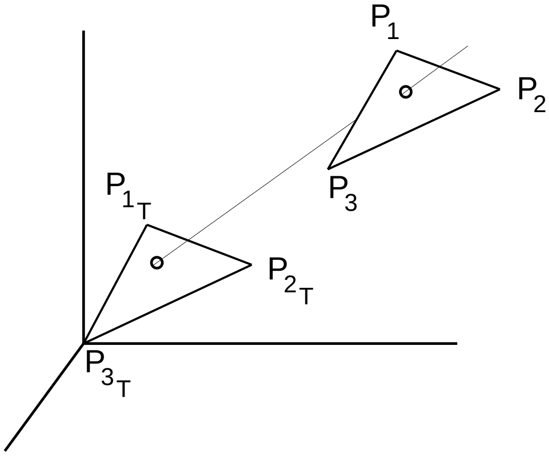


Figure 11.

For Method I or Method II. the centers found are "exploded" to the surface of the sphere using formula [9] and formula [10].

Using the coordinates, this program finds the lengths of the elements of the structure ($\ell$), the angle between pairs of elements (face angle $\alpha$). the angle between the elements and a radius from the origin to an endpoint of the element (axial angle $\omega$), and the angle between adjacent faces of the structure (dihedral angle $\beta$). Figure 12.

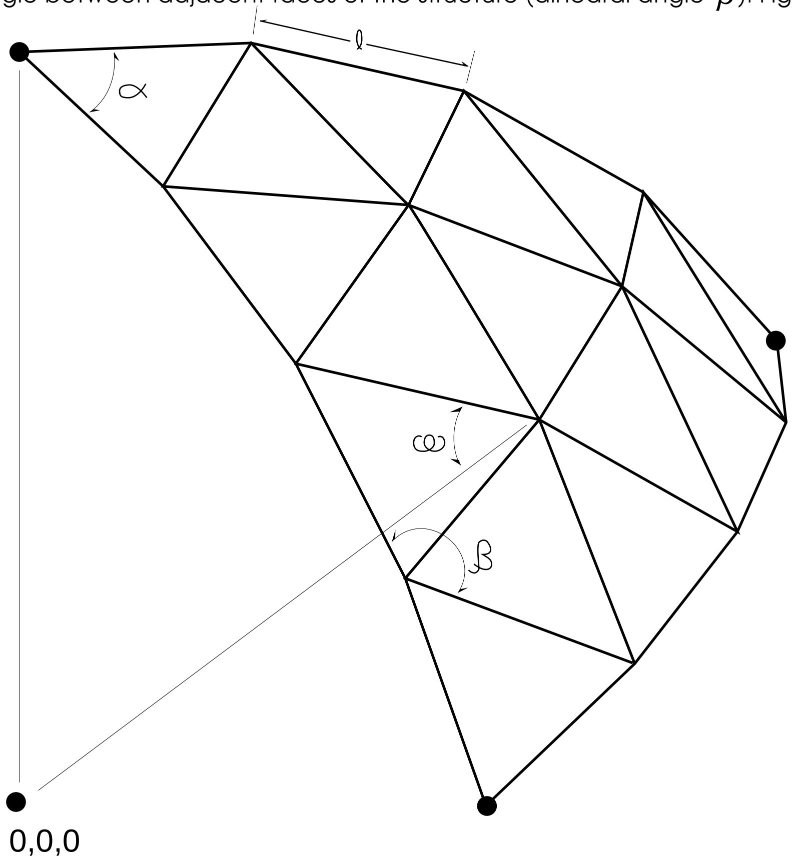


Figure 12.

To find the angle between elements the face $\angle \alpha$. we use the coordinates of their endpoints. The vertex of the angle is a common endpoint to each element and is translated to the origin. The other two endpoints $P_1$ and $P_2$ are translated in the same manner. Letting $(x_1\ y_1\ z_1)$ and $(x_2\ y_2\ z_2)$ be the points resulting from the translations of the endpoints $P_1$ and $P_2$,

$$\cos\alpha = \left|\frac{x_1x_2 + y_1y_2 + z_1z_2}{d_1d_2}\right|$$

$$\text{where } d_1 = \sqrt{x_1^{\,2} + y_1^{\,2} + z_1^{\,2}}$$

$$\text{and } d_2 = \sqrt{x_2^{\,2} + y_2^{\,2} + z_2^{\,2}}$$

$\alpha$ is the desired angle.

To find axial angles the above method is used except that the vertex is established at one end of an element and the origin is used with the other endpoint to define the angle. The desired angle is $\omega$.

The angle between two adjacent faces, the dihedral $\angle \beta$. is found using

$$\cos\beta = \frac{-\left|A_1A_2 + B_1B_2 + C_1C_2\right|}{\sqrt{A_1^{\,2} + B_1^{\,2} + C_1^{\,2}}\ \sqrt{A_2^{\,2} + B_2^{\,2} + C_2^{\,2}}}$$

where $\beta$ is the desired angle.

$A_1X + B_1Y + C_1Z + D_1 = 0$ defines the plane containing one face and
$A_2X + B_2Y + C_2Z + D_2 = 0$ defines the plane containing the other face.

The negative sign is used because the obtuse angle is desired.

The A, B, and C for each plane are computed as

$$A = \begin{vmatrix} Y_1 & Z_1 & 1 \\ Y_2 & Z_2 & 1 \\ Y_3 & Z_3 & 1 \end{vmatrix}$$

$$B = \begin{vmatrix} Y_1 & Z_1 & 1 \\ Y_2 & Z_2 & 1 \\ Y_3 & Z_3 & 1 \end{vmatrix}$$

$$C = \begin{vmatrix} Y_1 & Z_1 & 1 \\ Y_2 & Z_2 & 1 \\ Y_3 & Z_3 & 1 \end{vmatrix}$$

where $(X_1\ Y_1\ Z_1)$, $(X_2\ Y_2\ Z_2)$ and $(X_3\ Y_3\ Z_3)$ lie in the plane. In particular the three vertices of each face are used.

The length of the elements $\ell$ are found by using the general equation:

$$\ell = \sqrt{\left(P_{x_1} - P_{x_2}\right)^2 + \left(P_{y_1} - P_{y_2}\right)^2 + \left(P_{z_1} - P_{z_2}\right)^2}$$

where $\ell$ is the desired length.

To reduce total output, this program takes into account certain symmetries and outputs only a part of the total angles and lengths. The rest of the values are the same as at least one outputed value and can easily be found using the following symmetries, Figure 13.

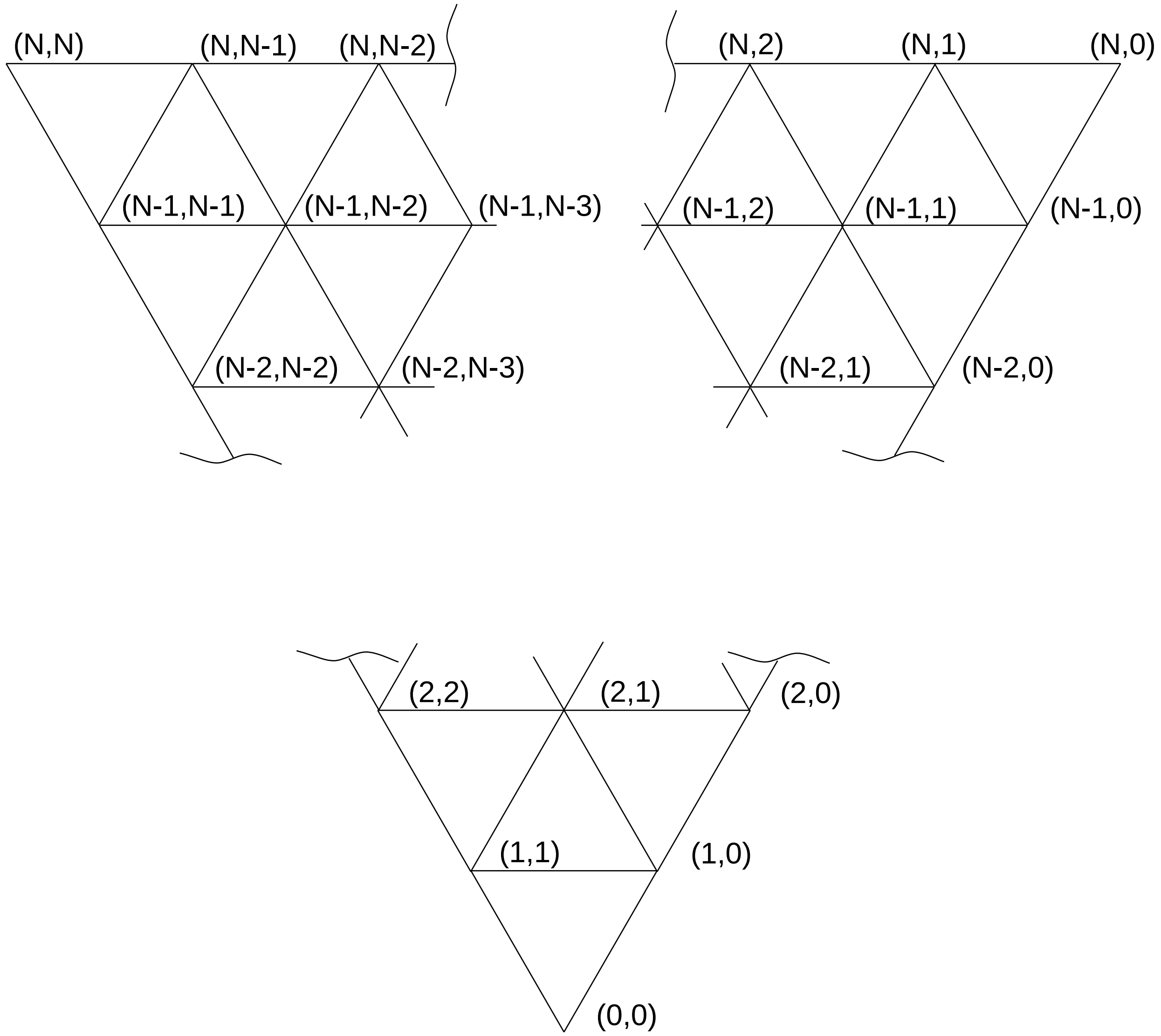


Figure 13.

FACE ANGLES

For every face angle opening directly towards (or away from) the point (0,0), there are equal angles opening towards (or away from) the point (N,0) and (N,N). for example, the angle (1,1), (0,0), (1,0) with vertex at (0,0) is equal to the angle (N - 1,0), (N,0), (N,1) and the angle (N,N - 1), (N,N), (N - 1, N - 1). Thus, only the face angles facing directly towards or away from (0,0) are computed. If the vertex is to lie at (I,J), the angle will be either (I + I, J + I), (I, J), (I + I, J) or (I - I, J - I), (I,J), (I - I, J). Also, only the face angles falling on the right of or on a line passing through $\left(X_1\ Y_1\ Z_1\right)$ and the midpoint of the opposite side are computed.

The elements of the structure can be put into one-to-one correspondence with the lengths and dihedral angles. The dihedral angle associated with an element is the angle between the two faces containing the element. For each element, there are two axial angles, one at each end, but since the element is a cord of the circle, the two angles are equal and may be considered one. In this case, we have a one-to-one correspondence between elements and axial angles. This program will only compute values around elements parallel to the side opposite $\left(X_1\ Y_1\ Z_1\right)$ and on the right side of a line through $\left(X_1\ Y_1\ Z_1\right)$ and the midpoint of the opposite side. All other lengths and angles are symmetric to one of the lengths and angles computed in this manner.

## PROJECTED RESEARCH

Another method is being developed of subdividing a polyhedron and "Exploding" it into a spherical form. A correlational study will be conducted to determine the difference in material usage, volume, and surface area of the subdivided forms, using the method described in this report, previously developed methods, methods presently under development, and the Fuller methods of subdividing a sphere.

A mathematical model is also being developed which will explain the transformation of polyhedral forms as expandable structures.

## APPENDIX I

The computer program here contained, was written for the IBM 7044 computer, utilizing FORTRAN language. The program may be used for a Tetrahedron, Octahedron, or Icosahedron, depending upon the coordinates used as input data. The output is given in units based upon a radius of 1 for the spherical form and therefore may be used as a basis for determining large structures.

The example of the output data given here is for a six frequency icosahedral sphere, and may be read as example I. The output takes advantage of symmetries within the spherical icosahedron as discussed in the text material. See Figure 14.

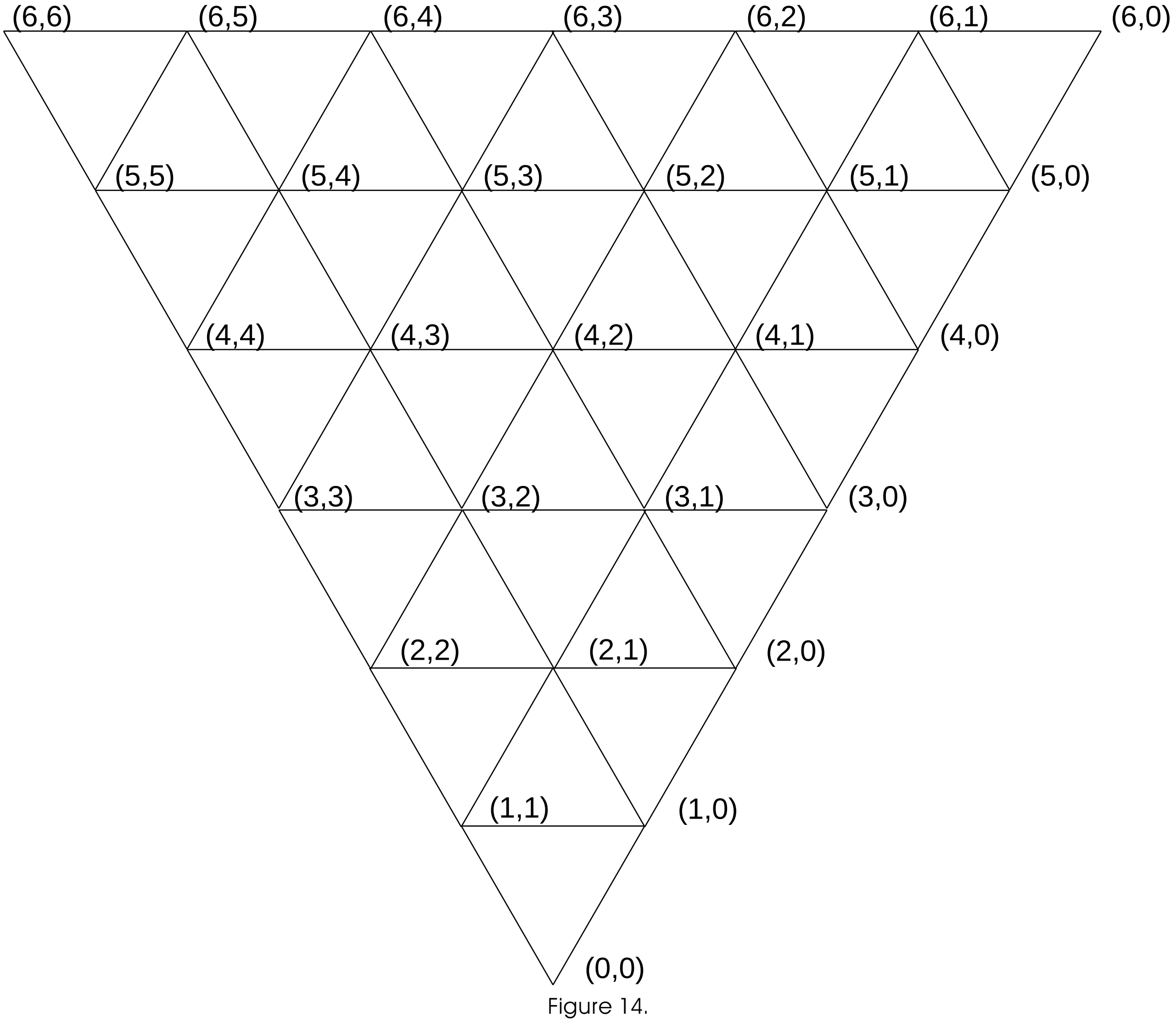


Figure 14.

CHART G1

B1 Read the parameters for a DO loop governing the frequencies and two variables; one indicating the basic structure (tetrahedron, octahedron, or icosahedron) and another indicating what method of division will be used.

C1-C2 If the card was blank (first parameter zero), stop.

D1 Determine from the card read, what coordinates are to be used as the basic structure and the coefficients of some equations. This information is stored in an array by a DATA statement.

E1 Enter a DO loop governing the frequency.

Set the summations for total area, length, and volume to zero. There are two summations of length: one for total structure and one above a face of the basic structure.

Write a leader record containing the name of the basic structure, the frequency, and the method of division.

H1-J1 Find and write the total number of faces, edges, and vertices both in the entire structure and above one face of the basic structure. These figures are used for display only and are not used in any further calculations.

K1 Enter DO loop to determine the first end point of the edge.

A3 Enter DO loop to determine the second end point of the edge.

B3 The end points of the edges should not be the same.

C3 Rotate the plane defined by the end points and the origin into the X-Y plane with one of the end points into the X axis, and calculate the angle $\phi = \text{Arctan}\left(y^2/x^2\right)$ necessary for subdivision.

D3 Enter DO loop to determine the subdivision of the edge.

E3 Generate points of subdivision along the edge by N subdivisions of angle $\phi$.

F3 Rotate the generated points back into the original plane.

G3 Store the points generated into their respective arrangement.

H3-K3-J3 If DO loop is complete there should be no more points on their respective edges.

A4 The three vertices of the basic structure are arbitrarily ordered. The first vertice is exploded to the surface of the unit sphere to explode a point. This is done by dividing the coordinates of a point by its distance from the origin.

B4 Print out the coordinate of this point: G2J1-G2K1; G2E2-G2F2; G2C3-G2D3.

C4 G1J5-G1K5; G2F1-G2G1; these blocks are used to count lines orinted and to provide margins at the top and bottom of each page.

D4 Subdivision of the basic face will be generated by using the coordinates stored in SD1, SD2, and SD3 and taking all lines parallel to their respective edges. The intersections of these lines creates windows of which their coordinates are necessary for finding their centers (two methods of finding the windows center are used) for completion of the subdivision. Enter DO loop which generates points along the edge from the first vertex to the second.

E3 Enter a DO loop which generates points along the edge from the first vertex to the second. These points will be the first in each row of points parallel to the edge from the second vertex to the third. The first row will be the one nearest the first vertex.

E4 Enter DO loop which generates the points along the Tow determined in E4.

F4 Determine the I. J. values of the basic unit.

G4 Enter DO loop which generates the coordinates of the basic unit.

H4 Set up constants of points in basic units used in determining coordinates of the windows.

J4-K4 All points should be within the PIT. If not, set equal to zero (necessary for finding dihedral angles along PIT edges.)

A5-BS Points along the PIT edges may be found directly from an SD array

C5 INTERC is a subroutine that finds the coordinates of the intersection of 2 lines; used to find the coordinates of the vertice of the window

D5 Determine the center of the window by one of two methods: Method #1 find the center by taking the intersection of four lines, each passing through a vertice of the window and the midpoints of the opposites side

E5 This is the end of the DO loop that finds all of the points within the basic unit.

F5 This explodes all points of the basic unit to the surface of the sphere.

H5 Writes the coordinates of $P_1$ and $P_3$

CHART G2

B1 Due to symmetry, not all angles and lengths are needed.

C1 Find the length between $P_1$ and $P_2$.

D1 Add a multiple of this length into the sums. A multiple is used because other elements symmetric to it will not be found due to the conditions of B1.

E1 Lable and write this length.

H1 Find the axial angle with vertex angle at $P_1$ and legs through $P_3$ and the origin.

J1 Lable and write this axial angle.

B2 Find the total area of and volume under the face generated by $P_1$, $P_2$, and $P_3$.

C2 Add this into the summations.

D2 Find the face angles with vertices at $P_2$ and $P_4$ and legs passing through $P_1$ and $P_3$.

E2 Lable and write the face angles.

H2-K2 If $P_1$ is in the last row or the face of the basic structure, then there is not face between $P_1$, $P_3$, and $P_4$. Otherwise, the area of and volume under that face will be added into the summations.

A3 Find the dihedral angle between the two faces formed by $P_1$, $P_2$, $P_3$, and $P_4$. If $P_4$ is not above this face of the basic structure, it will not be properly generated. In this case, find the angle between the face formed by $P_1$, $P_2$, and $P_3$, and a plane passing through $P_1$, $P_3$, and the origin, doubling this result.

B3 Lable and write this dihedral angle.

E3 This is the end of the DO loop of block G1E4. If the loop hasn't been satisfied, the next point will be the first in the next row.

G3 This is the end of DO loop of block G1D4. If the loop hasn't been satisfied, the next point will be the first in the next row.

H3-J3 Find, lable, and write the total areas, volumes, and lengths of the structure.

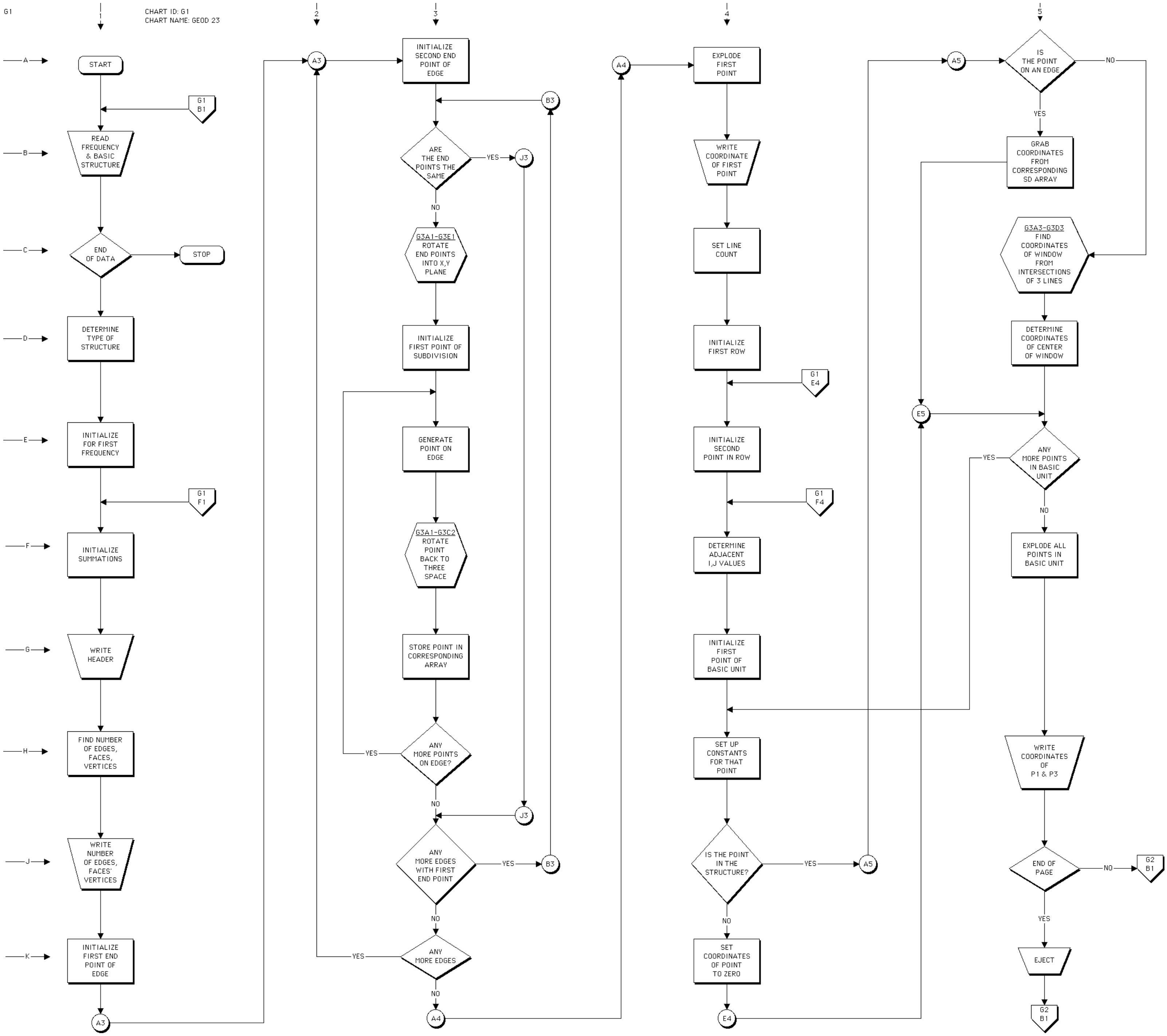
K3 This is the end of the DO loop of block G1E1. If the loop hasn't been satisfied, the same basic structure will be used for the next frequency. Otherwise a new card is read.

As used in blocks G1F1, G1H1-G1J1:

the vertices above a face are those vertices exploded fro. points on that face. The lengths and areas above the face are those created by the vertices above the face.

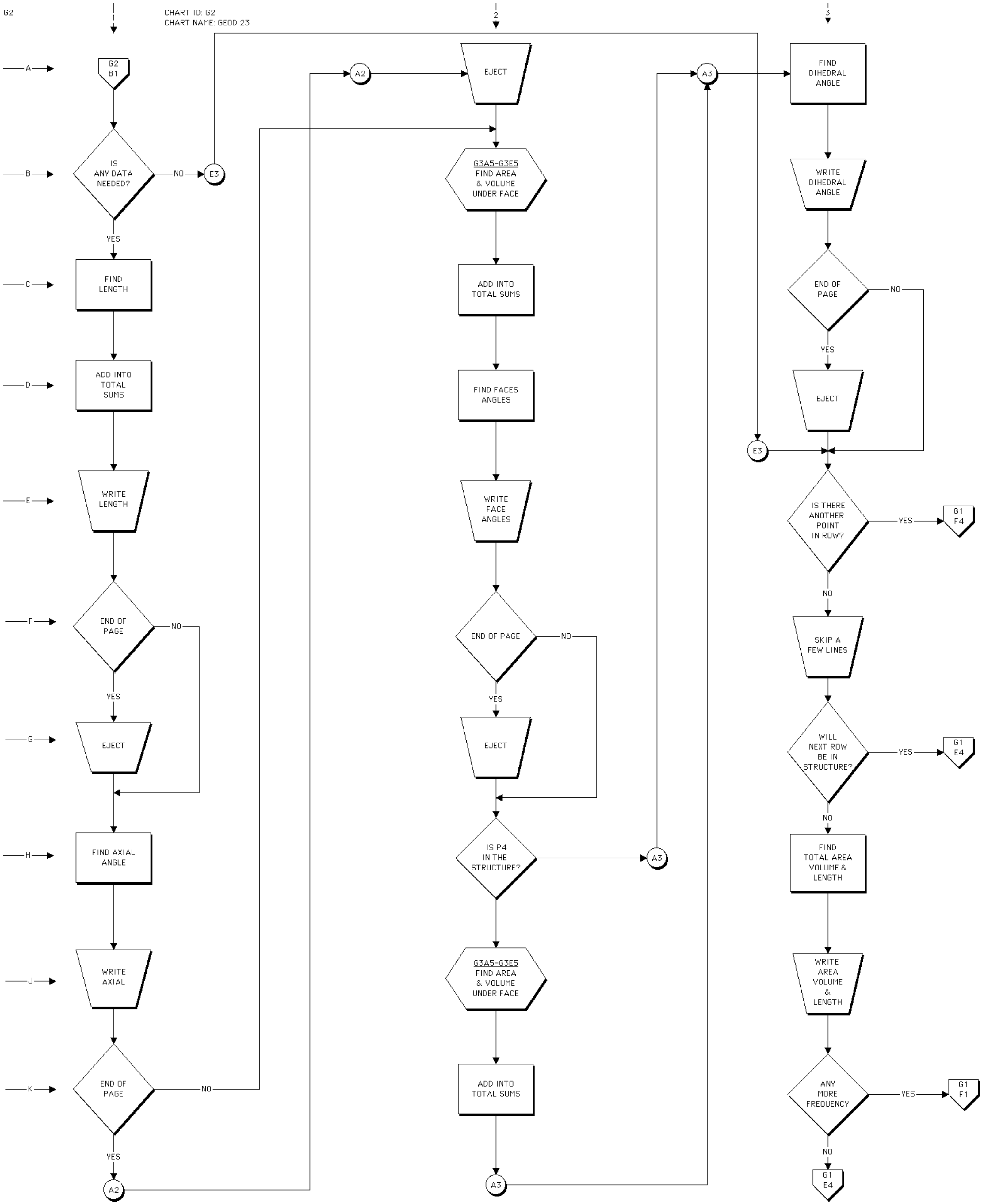
As used in blocks G1H1-G1J1, G1E5:

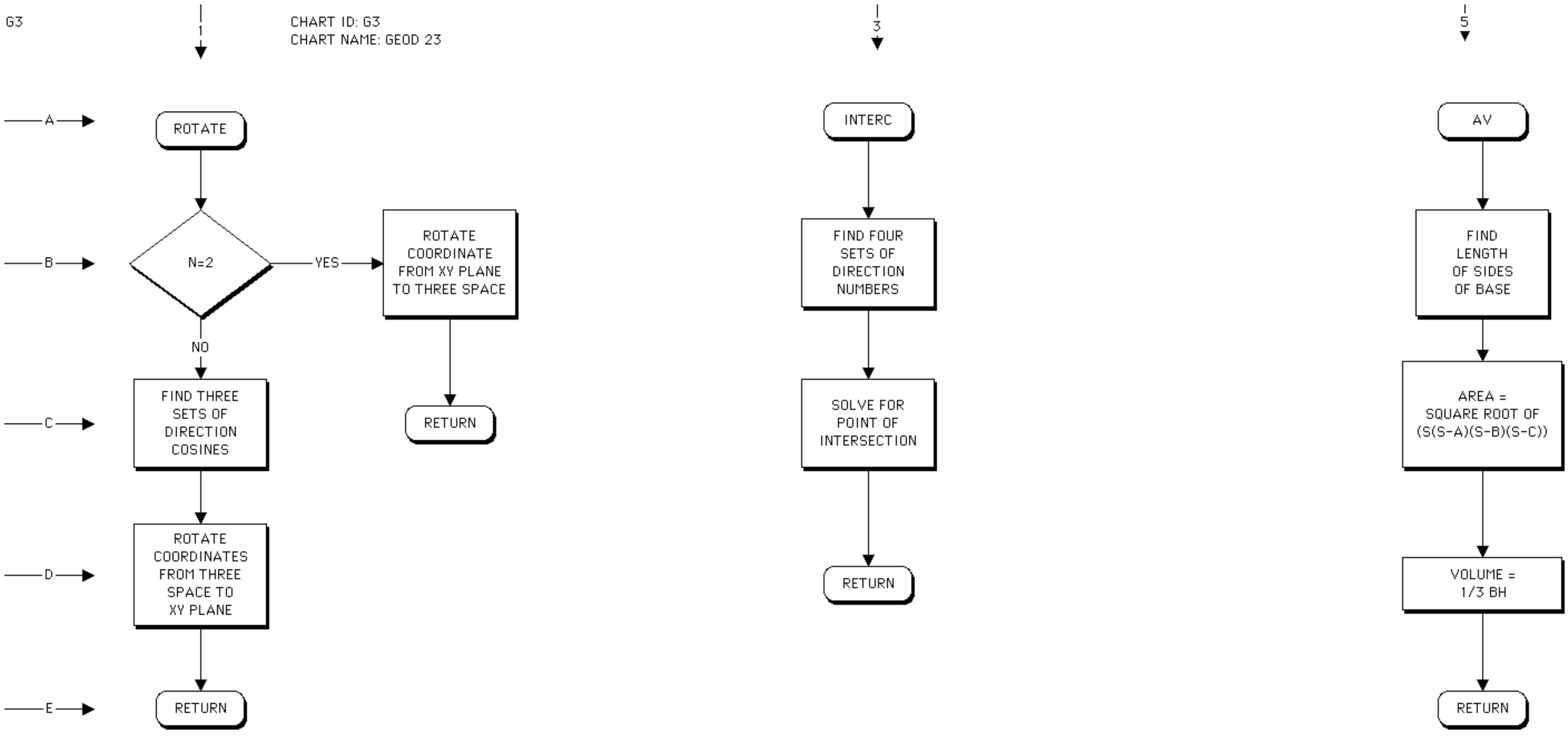
the basic unit consists of four points combined in two adjacent triangles with two parts approximately parallel to the edge containing vertex 2 and 3.


G1
CHART ID: G1
CHART NAME: GEOD 23
1
2
3
4
5
A
B
C
D
E
F
G
H
J
K
START
G1 B1
READ FREQUENCY & BASIC STRUCTURE
END OF DATA
STOP
DETERMINE TYPE OF STRUCTURE
INITIALIZE FOR FIRST FREQUENCY
G1 F1
INITIALIZE SUMMATIONS
WRITE HEADER
FIND NUMBER OF EDGES, FACES, VERTICES
WRITE NUMBER OF EDGES, FACES' VERTICES
INITIALIZE FIRST END POINT OF EDGE
A3
A3
INITIALIZE SECOND END POINT OF EDGE
B3
ARE THE END POINTS THE SAME
YES
J3
NO
G3A1-G3E1 ROTATE END POINTS INTO X,Y PLANE
INITIALIZE FIRST POINT OF SUBDIVISION
GENERATE POINT ON EDGE
G3A1-G3C2 ROTATE POINT BACK TO THREE SPACE
STORE POINT IN CORRESPONDING ARRAY
ANY MORE POINTS ON EDGE?
YES
NO
J3
ANY MORE EDGES WITH FIRST END POINT
YES
B3
NO
ANY MORE EDGES
YES
NO
A4
A4
EXPLODE FIRST POINT
WRITE COORDINATE OF FIRST POINT
SET LINE COUNT
INITIALIZE FIRST ROW
G1 E4
INITIALIZE SECOND POINT IN ROW
G1 F4
DETERMINE ADJACENT I,J VALUES
INITIALIZE FIRST POINT OF BASIC UNIT
SET UP CONSTANTS FOR THAT POINT
IS THE POINT IN THE STRUCTURE?
YES
A5
NO
SET COORDINATES OF POINT TO ZERO
E4
A5
IS THE POINT ON AN EDGE
NO
YES
GRAB COORDINATES FROM CORRESPONDING SD ARRAY
G3A3-G3D3 FIND COORDINATES OF WINDOW FROM INTERSECTIONS OF 3 LINES
DETERMINE COORDINATES OF CENTER OF WINDOW
E5
ANY MORE POINTS IN BASIC UNIT
YES
NO
EXPLODE ALL POINTS IN BASIC UNIT
WRITE COORDINATES OF P1 & P3
END OF PAGE
NO
G2 B1
YES
EJECT
G2 B1

G2

CHART ID: G2
CHART NAME: GEOD 23




G3
1
CHART ID: G3
CHART NAME: GEOD 23
3
5
A
B
C
D
E
ROTATE
N=2
YES
NO
ROTATE COORDINATE FROM XY PLANE TO THREE SPACE
RETURN
FIND THREE SETS OF DIRECTION COSINES
ROTATE COORDINATES FROM THREE SPACE TO XY PLANE
RETURN
INTERC
FIND FOUR SETS OF DIRECTION NUMBERS
SOLVE FOR POINT OF INTERSECTION
RETURN
AV
FIND LENGTH OF SIDES OF BASE
AREA = SQUARE ROOT OF (S(S-A)(S-B)(S-C))
VOLUME = 1/3 BH
RETURN

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C        1         2         3         4         5         6         7         8
C     78901234567890123456789012345678901234567890123456789012345678901234567890
$IBFTC GEOD NODECK
      DIMENSION SD 1(3, 32), SD 2(3, 32), SD 3(3, 32) , P(4, 3)
      DIMENSION COORD(3, 3, 3), V(3, 3), LAB(2, 4), NAMES(2, 3),
     1 FIG(2, 3)
      INTEGER EFV(3, 3), PAG SIZ, TE OC IC, VL, EL, FL, VG, EG, FG
      REAL L OF TRI, LENGTH, L CF FIG
      DATA FIG / 4., 8., 8., 16., 20., 40. /
      DATA NAMES / 12H TETRAHEDRON , 12H OCTAHEDRON , 12H ICOSAHEDRON /,
     1 PAG SIZ / 56 /
      DATA COORD / -.57735027, -.57735027. .57735021. .57735027,
     1 -.57735027, -.57735027, -.57735027, .57735027. -.57735027,
     2 1., 0., 0., 0., 0., 1., 0., 1., 0., 0., .85065081, .52573111,
     3 .85065081, .52573111, 0., .52573111, 0., .85065081 /
      DATA EFV / 4, 8, 20, 2, 4, 10, 6, 12, 30 /
      EQUIVALENCE
     1 (V(1, 1), X1), (V(2, 1), Y1), (V(3, 1), Z1),
     2 (V(1, 2). X2), (V(2, 2), Y2), (V(3, 2), Z2),
     3 (V(1, 3), X3), (V(2, 3), Y3), (V(3, 3), Z3),
     4 (LAB(1, 1), I1), (LAB(2, 1), J1), (LAB(1, 2), I2),
     5 (LAB(2, 2), J2), (LAB(1, 3), I3), (LAB(2, 3), J3),
     6 (LAB(l, 4), I4), (LAB(2, 4), J4)
      COEF(A1, A2, A3, B1, B2, B3) = A1 * (B2 - B3) - A2 * (B1 - B3) +
     1 A3 * (B1 - B2)
   70 FORMAT (412)
   71 FORMAT (10H1FREQUENCY 15, 2X, 2A6, 5X, 8HMETHOD 2)
   72 FORMAT (7H0V(L) = I5, 9H E(L) = I5, 9H F(L) = I5, 9H V(G) =
     * 15, 9H   E(G) = I5, 9H F(G) = I5 ///)
   73 FORMAT (9HL 0 0 3(5X, F10.6)///)
   74 FORMAT (1HL 214, 3(5X, F10.6))
   75 FORMAT (lHA)
   76 FORMAT (9X, 8HLENGTH  , 2(3X, 214), 21X, F11.8)
   77 FORMAT (9X, 8HAXIAL   , 11H 0.0 0.0, 2(3X, 214), 10X, 2F11.6)
   78 FORMAT (9X, 8HFACE    , 3(3X, 214), 10X, 2F11.6)
   79 FORMAT (9X, 8HDIHEDRAL , 2(3X, 214), 21X, 2F11.6)
   80 FORMAT (//)
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81 FORMAT (12H1 ONE FACE/ 9X, 4HAREA, 27X, F15.6, / 9X, 6HVOLUME,
  * 25X, F15.6 / 9X. 6HLENGTH, 25X. F15.6 // 22H COMPLETE STRUCTURE
  * / 9X, 4HAREA, 27X, F15.6 / 9X, 6HVOLUME, 25X, F15.6, / 9X,
  * 6HLENGTH, 25X, F15.6)
 1 READ (5, 70) MIN, MAX, INC, TE OC IC
   IF (MIN .EQ. 0) STOP
   IF (MAX .EQ. 0) MAX = MIN
   IF (INC .EQ. 0) INC = 1
   K = 1
   IF (TE OC IC .EQ. 8) K = 2
   IF (TE OC IC .EQ. 20) K = 3
   DO 2 I = 1, 3
   DO 2 J = 1, 3
 2 V(J, I) = COORD(J, I, K)
   DO 25 N = MIN, MAX, INC
   FLOAT N = N
   SUM OF L = 0.
   SUM OF A = 0.
   SUM OF V = 0.
   L OF TRI = 0.
   WRITE (6, 71) N, NAMES(1, K), NAMES(2, K)
   VL = (N + 1) * (N + 2) / 2
   EL = 3 * N * (N + 1) / 2
   FL = N**2
   VG = EFV (K, 2) * N**2 + 2
   EG = EFV (K, 3) * N**2
   FG = EFV (K, 1) * FL
   WRITE (6, 72) VL, EL, FL, VG, EG, FG
   L1 = 0
   DO 5 L2 = 1, 2
   DO 5 LN = 2, 3
   IF (L2 .EQ. LN) GO TO 5
   L1 = L1 + 1
   CALL ROTATE (1, V(1, LN), V(2, LN), V(3, LN), X2R, Y2R, Y2R, Z2R,
  1 V(1, L2), V(2, L2), V(3, L2))
   THETA = ATAN2(Y2R, X2R)
   DO 5 L4 = 1, N
   T = L4
   ANG = T * THETA / FLOAT N
   X = COS(ANG)
   Y = SIN(ANG)
   A1 = -Y
   B1 = X
   A2 = -Y2R
   B2 = X2R - 1.
   C2 = B2
   Y = (A1*C2) / (A1*B2 - A2*B1)
   X = (B1*C2) / (A2*B1 - A1*B2)
   CALL ROTATE (2, X, Y, O., XR, YR, ZR)
   IF (L1 - 2) 27, 3, 4
27 SD 1(1, L4) = XR
   SD 1(2, L4) = YR
   SD 1(3, L4) = ZR
   GO TO 5
 3 SD 2(1, L4) = XR
   SD 2(2, L4) = YR
   SD 2(3, L4) = ZR
   GO TO 5
 4 SD 3(1, L4) = XR
   SD 3(2, L4) = YR
```

```
      SD 3(3, L4) = ZR
    5 CONTINUE
      R = SQRT(X1**2 + Y1**2 + Z1**2)
      P(1, 1) = X1 / R
      P(1, 2) = Y1 / R
      P(1, 3) = Z1 / R
      WRITE (6, 73) P(1, 1), P(1, 2), P(1, 3)
      LINES = 12
      DO 24 I1 = 1, N
      DO 23 J1 = 1, I1
      I2 = I1 - 1
      J2 = Jl - l
      I3 = I1
      J3 = J2
      I4 = I1 + 1
      J4 = J1
      DO 11 LA = 1, 4
      LI = LAB(1, LA)
      LJ = LAB(2, LA)
      IJ = LI - LJ
      NJ = N - LJ
      IF (LI .LE. N) GO TO 65
      P(LA, 1) = 0.
      P(LA, 2) = 0.
      P(LA, 3) = 0.
      GO TO 11
   65 IF (LI .EQ. LJ) GO TO 7
      IF (LJ .EO. 0) GO TO 9
      IF (LI .NE. N) GO TO 10
      P(LA, 1) = SD 3(1, NJ)
      P(LA, 2) = SD 3(2, NJ)
      P(LA, 3) = SD 3(3, NJ)
      GO TO 11
    7 IF (LJ .EQ. 0) GO TO 8
      P(LA, 1) = SD 1(1, LI)
      P(LA, 2) = SD 1(2, LI)
      P(LA, 3) = SD 1(3, LI)
      GO TO 11
    8 P(LA, 1) = X1
      P(LA, 2) = Y1
      P(LA, 3) = Z1
      GO TO 11
    9 P(LA, 1) = SD 2(1, LI)
      P(LA, 2) = SD 2(2, LI)
      P(LA, 3) = SD 2(3, LI)
      GO TO 11
   10 CALL INTERC (SD 1(1, LI), SD 1(2, LI), SD 1(3, LI), SD2(1, LI),
     1 SD 2(2, LI), SD 2(3, LI), SD 3(1, NJ), SD 3(2, NJ), SD 3(3, NJ),
     2 SD 1(1, LJ), SD 1(2, LJ), SD 1(3, LJ), XE1, YE1, ZE1)
      CALL INTERC (SD 3(1, IJ), SD 3(2, IJ), SD 3(3, IJ), SD 2(1, IJ),
     1 SD 2(2, IJ), SD 2(3, IJ), SD 3(1, NJ), SD 3(2, NJ), SD 3(3, NJ),
     2 SD 1(1, LJ), SD 1(2, LJ ), SD 1(3, LJ), XE2, YE2, ZE2)
      CALL INTERC (SD 3(1, IJ), SD 3(2, IJ), SD 3(3, IJ), SD 2(1, IJ),
     1 SD 2(2, IJ), SD 2(3, IJ), SD 1(1, LI), SD 1(2, LI), SD 1(3, LI),
     2 SD 2(1, LI), SD 2(2, LI), SD 2(3, LI), XE3, YE3, ZE3)
C      BEGIN DETERMINATION OF CENTER OF WINDON IN PLANE OF MAJOR FACE . . .
       P(LA, 1) = (XE1 + XE2 + XE3) / 3.
       P(LA, 2) = (YE1 + YE2 + YE3) / 3.
       P(LA, 3) = (ZE1 + ZE2 + ZE3) / 3.
    11 CONTINUE
    L = 4
    IF (I4 .GT. N) L = 3
    DO 12 INDEX 1 = 1, L
    R = SQRT(P(INDEX 1, 1)**2 + P(INDEX 1, 2)**2 + P(INDEX 1, 3)**2)
    DO 12 INDEX 2 = 1, 3
 12 P(INDEX 1, INDEX 2) = P(INDEX 1, INDEX 2) / R
    IF (J3 .NE. 0) GO TO 125
    WRITE (6, 74) I3, J3, P(3, 1), P(3, 2), P(3, 3)
    LINES = LINES + 3
    IF (LINES .LT. PAG SIZ) GO TO 125
    WRITE(6, 75)
    LINES = 1
125 WRITE (6, 74) I1, J1, P(1, 1), P(1, 2), P(1, 3)
    LINES = LINES + 3
    IF (LINES .LT. PAG SIZ) GO TO 13
    WRITE (6, 75)
    LINES = 1
 13 IF (2 * J3 .GE. I3) GO TO 23
    LENGTH = SQRT((P(1, 1) - P(3, 1))**2 + (P(1, 2) - P(3, 2))**2 +
   * (P(1, 3) - P(3, 3))**2)
    L OF TRI = L OF TRI + 3. * LENGTH
    IF (2 * J1 .GI. I1) GO TO 14
    L OF TRI = L OF TRI + 3. * LENGTH
    SUM OF L = SUM OF L + 1.5 * LENGTH
    IF (I1 .LT. N) SUM OF L = SUM OF L + 1.5 * LENGTH
    GO TO 15
 14 SUM OF L = SUM OF L + .75 * LENGTH
    IF (I1 .LT. N) SUM OF L = SUM OF L + .75 * LENGTH
 15 WRITE (6, 76) I1, J1, I3, J3, LENGTH
    LINES = LINES + 1
    IF (LINES .LT. PAG SIZ) GO TO 16
    WRITE (6, 75)
    LINES = 1
 l6 XT = P(1, 1) - P(3, 1)
    YT = P(1, 2) - P(3, 2)
    ZT = P(1, 3) - P(3, 3)
    D1D2 = SQRT(XT**2 + YT**2 + ZT**2)
    ANG = ARCOS(ABS(XT*P(3, 1) + YT*P(3, 2) + ZT*P(3, 3))/ D1D2)
    DEG = ANG / .017453293
    WRITE (6, 77) I3, J3, I1, J1, DEG, ANG
    LINES = LINES + 1
    IF (LINES .LT. PAG SIZ) GO TO 17
    WRITE (6, 75)
    LINES = 1
 17 CALL AV(P(1, 1), P(1, 2), P(1, 3), P(3, 1), P(3, 2), P(3, 3),
   * P(2, 1), P(2, 2), P(2, 3), AREA, VOLUME)
    IF (2 * J1 .LE. I1) GO TO 18
    AREA = .5 * AREA
    VOLUME = .5 * V0LUME
 18 SUM OF A = SUM OF A + AREA
    SUM OF V = SUM OF V + VOLUME
    DO 19 INDEX 1 = 2, 4, 2
    XA - P(1, 1) - P(INDEX 1, 1)
    YA - P(1, 2) - P(INDEX 1, 2)
    ZA - P(1, 3) - P(INDEX 1, 3)
    XB - P(3, 1) - P(INDEX 1, 1)
    YB - P(3, 2) - P(INDEX 1, 2)
    ZB - P(3, 3) - P(INDEX 1, 3)
    D1D2 = SQRT((XA**2 + YA**2 + ZA**2)) * SQRT((XB**2 + YB**2 +
   * ZB**2))
    ANG =  ARCOS((XA*XB + YA*YB + ZA*ZB) / D1D2)
    DEG = ANG / .017453253
    WRITE (6, 78) I1, J1, LAB(1, INDEX 1), LAB(2, INDEX, 1), I3, J3,
   1 DEG, ANG
    LINES = LINES + 1
    IF (LINES .LT. PAG SIZ) GO TO 19
    WRITE (6, 75)
    LINES = 1
 19 IF (I4 .GT. N) GO TO 21
    CALL AV(P(1, 1), P(1, 2), P(1, 3), P(3, 1), P(3, 2), P(3, 3),
   * P(4, 1), P(4, 2), P(4, 3), AREA, VOLUME)
    IF (2 * J1 .LE. I1) GO TO 20
    AREA  = .5 * AREA
    VOLUME = .5 * VOLUME
 20 SUM OF A = SUM OF A + AREA
    SUM OF V = SUM OF V + VOLUME
 21 A1 = COEF(P(1, 2), P(2, 2), P(3, 2), P(1, 3), P(2, 3), P(3, 3))
    B1 = COEF(P(1, 1), P(2, 1), P(3, 1), P(1, 3), P(2, 3), P(3, 3))
    C1 = COEF(P(1, 1), P(2, 1), P(3, 1), P(1, 2), P(2, 2), P(3, 2))
    A2 = COEF(P(1, 2), P(4, 2), P(3, 2), P(1, 3), P(4, 3), P(3, 3))
    B2 = COEF(P(1, 1), P(4, 1), P(3, 1), P(1, 3), P(4, 3), P(3, 3))
    C2 = COEF(P(1, 1), P(4, 1), P(3, 1), P(1, 2), P(4, 2), P(3, 2))
    D1D2 = SQRT((A1**2 + B1**2 + C1**2) * (A2**2 + B2**2 + C2 **2))
    ANG = ARCOS(-ABS(A1*A2 + B1*B2 + C1*C2) / D1D2)
    IF (I4 .GT. N) ANG = 2. * (3.1415927 - ANG)
    DEG - ANG / .017453293
    WRITE (6, 79) I1, J1, I3, J3, DEG, ANG
    LINES = LINES + 1
    IF (LINES .LT. PAG SIZ) GO TO 23
    WRITE (6, 75)
    LINES = 1
 23 CONTINUE
    WRITE (6, 80)
    LINES = LINES + 3
    IF (LINES .LT PAG SIZ) GO TO 24
    WRITE (6, 75)
    LINES = 1
 24 CONTINUE
    A OF TRI = 2. * SUM OF A
    V OF TRI = 2. * SUM OF V
    A OF FIG = FIG(1, K) * A OF TRI
    V OF FIG = FIG(1, K) * V OF TRI
    L OF FIG = FIG(2, K) * SUM OF L
    WRITE (6, 81) A OF TRI, V OF TRI, L OF TRI, A OF FIG, V OF FIG,
   * L OF FIG
 25 CONTINUE
    GO TO 1
    END
```

```
$IBFTC ROTATE NODECK
       SUBROUTINE ROTATE (N, X2, Y2, Z2, X2R, Y2R, Z2R, X1, Y1, Z1)
       IF (N .EQ. 2) GO TO 1
       D1 = SQRT(X1**2 + Y1**2 + Z1**2)
       H1 = X1 / D1
       U1 = Y1 / D1
       V1 = Z1 / D1
       H3 = Y1 * Z2 - Y2 * Z1
       U3 = Z1 * X2 - X1 * Z2
       V3 = X1 * Y2 - X2 * Y1
       D2 = SQRT(H3**2 + U3**2 + Y3**2)
       H3 = H3 / D2
       U3 = U3 / D2
       V3 = V3 / D2
       H2 = U3 * Z1 - Y1 * V3
       U2 = V3 * X1 - Z1 * H3
       V2 = H3 * Y1 - X1 * U3
       D3 = SQRT(H2**2 + U2**2 + V2**2)
       H2 = H2 / D3
       U2 = U2 / D3
       V2 = V2 / D3
       X2R = X2 * H1 + Y2 * U1 + Z2 * V1
       Y2R = X2 * H2 + Y2 * U2 + Z2 * V2
       RETURN
     1 X2R = X2 * H1 + Y2 * H2 + Z2 * H3
       Y2R = X2 * U1 + Y2 * U2 + Z2 * U3
       Z2R = X2 * V1 + Y2 * V2 + Z2 * V3
       RETURN
       END
```

```
$IBFTC INTERC NODECK
       SUBROUTINE INTERC (XI, Y1, Z1, X2, Y2, Z2, X3, Y3, Z3, X4, Y4, Z4,
      * X, Y, Z)
       A1 = Y2 - Y1
       B1 = X1 - X2
       C1 = X1*(Y1 - Y2) + Y1*(X2 - X1)
       A2 = Y4 - Y3
       B2 = X3 - X4
       C2 = X3*(Y3 - Y4) + Y3*(X4 - X3)
       A3 = A1
       B3 = Z1 - Z2
       C3 = Z1*(Y1 - Y2) + Y1*(Z2 - Z1)
       A4 = A2
       B4 = Z3 - Z4
       C4 = Z3*(Y3 - Y4) + Y3*(Z4 - Z3)
       X = (B2*C1 - B1*C2) / (A2*B1 - A1*B2)
       Y = (A2*C1 - A1*C2) / (A1*B2 - A2*B1)
       Z = (B4*C3 - B3*C4) / (A4*B3 - A3*B4)
       RETURN
       END
```

```
$IBFTC AV NODECK
       SUBROUTINE AV (X1, Y1, Z1, X2, Y2, Z2, X3, Y3, Z3, AREA, VOLUME)
       X12 = X1 - X2
       X13 = X1 - X3
       X23 = X2 - X3
       Y12 = Y1 - Y2
       Y13 = Y1 - Y3
       Y23 = Y2 - Y3
       Z12 = Z1 - Z2
       Zl3 = Z1 - Z3
       Z23 = Z2 - Z3
       A = SQRT(X12**2 + Y12**2 + Z12**2)
       B = SQRT(X13**2 + Y19**2 + Z13**2)
       C = SQRT(X23**2 + Y23**2 + Z23**2)
       S = (A + B + C) / 2.
       AREA = SQRT(S*(S-A)*(S-B)*(S-C))
       A = Y1 * Z23 - Y2 * Z13 + Y3 * Z12
       B = X1 * Z23 - X2 * Z13 + X3 * Z12
       C = X1 * Y23 - X2 * Y13 + X3 * Y12
       D = X1 * (Y2*Z3 - Y3*Z2) - X2 * (Y1*Z3 - Y3*Z1) +
      * X3 * (Y1*Z2 - Y2*Z1)
       VOLUME = AREA * ABS(C) / SQRT(A**2 + B**2 + C**2) / 3.
       RETURN
       END
```

**Identification of Polyhedral Subdivision**
**Identification of Subdivision Method**

Frequency 6 Icosahedron  Method 2

| **No. o f vertices for PIT** | **No. of edges for PIT** | **No. of faces for PIT** | **Do. of vertices for total sphere** | **No. of edges for total sphere** | **Do. of faces for total sphere** |
|---|---|---|---|---|---|
| V(L) = 28 | E(L) = 63 | F(L) = 36 | V(G) = 362 | E(6) = 1080 | F(G) = 720 |

| **Vertex Identification** | **x coordinate** | **y coordinate** | **z coordinate** |
|---|---|---|---|
| 0  0 | 0.000000 | 0.850651 | 0.525731 |

| **Vertex Identification** | **x coordinate** | **y coordinate** | **z coordinate** |
|---|---|---|---|
| 1  0 | 0.107846 | 0.758171 | 0.643075 |

| **Vertex Identification** | **x coordinate** | **y coordinate** | **z coordinate** |
|---|---|---|---|
| 1  1 | 0.174499 | 0.66018 | 0.468576 |

| **= length of element** | **Element Identification** | | **Unit of length** |
|---|---|---|---|
| Length | 1  1 | 1  0 | 0.21569298 |

| | **angle identification** | | | | |
|---|---|---|---|---|---|
| **angle $\omega$** | **end pt.** | **vertex** | **end pt** | **degrees** | **radians** |
| Axial | 0.0  0.0 | 1 0 | 1 1 | 83.808804 | 1.462740 |

|  | angle identification |  |  |  |  |
|---|---|---|---|---|---|
| angle $\alpha$ | end pt. | vertex | end pt. | degrees | radians |
| Face | 1 1 | 0 0 | 1 0 | 71.646295 | 1.250464 |

|  | angle identification |  |  |  |  |
|---|---|---|---|---|---|
| angle $\alpha$ | end pt. | vertex | end pt. | degrees | radians |
| Face | 1 1 | 2 1 | 1 0 | 67.513278 | I.178329 |

|  | element identification |  |  |
|---|---|---|---|
| angle $\beta$ | dihedral edge | degrees | radians |
| Dihedral | 1 1 1 0 | 67.513278 | I.178329 |

**area, volume, length**
**for one PI?**

One Face

|  |  |
|---|---|
|  | **unit area** |
| **Area** | 0.623048 |
|  | **unit volume** |
| **Volume** | 0.206274 |
|  | **unit length of elements** |
| **Length** | 12.489358 |

**area, volume, length**
**for total sphere**

Complete Structure

|  |  |
|---|---|
|  | **total area** |
| **Area** | 12.460959 |
|  | **total volume** |
| **Volume** | 4.125473 |
|  | **total length of elements** |
| **Length** | 216.619799 |

```
FREQUENCY 6        ICOSAHEDRON       MHETHOD 2

V(L) = 28
E(L) = 63
F(L) = 36
V(G) = 362
E(G) = 1080
F(C) = 720

0   0        0.000000  0.850651    0.525731
1   0        0.107846  0.758171    0.643075
1   1        0.174499  0.866018    0.468576

LENGTH          1    1       1   0                           0.21569298
AXIAL         0.0  0.0       1   0       1   1              83.808804    1.462740
FACE            1    1       0   0       1   0              71.646295    1.250464
FACE            1    1       2   1       1   0              67.513278    1.178329
DIHEDRAL        1    1       1   0                         175.363857    3.060677
```

```
2    0        0.212031         0.639950          0.738585
2    1        0.291736         0.759807          0.581020

LENGTH          2    1        2   0                          0.21341332
AXIAL         0.0  0.0        2   0        2   1           83.874495    1.463886
FACE            2    1        1   0        2   0           68.618366    1.197616
FACE            2    1        3   1        2   0           64.858814    1.132000
DIHEDRAL        2    1        2   0                       174.704821    3.049174

2    2        0.343074         0.851981          0.395511
3    0        0.309017         0.500000          0.809017
3    1        0.401810         0.624919          0.669347

LENGTH          3    1        3   0                          0.20910066
AXIAL         0.0  0.0        3   0        3   1           83.998736    1.466055
FACE            3    1        2   0        3   0           65.240479    1.138661
FACE            3    1        4   1        3   0           62.105942    1.083953
DIHEDRAL        3    1        3   0                       174.030888    3.037412

3    2        0.466634         0.729806          0.499636

LENGTH          3    2        3   1                          0.20977452
AXIAL         0.0  0.0        3   1        3   2           83.979328    1.465716
FACE            3    2        2   1        3   1           64.995155    1.134380
FACE            3    2        4   2        3   1           61.944237    1.081131
DIHEDRAL        3    2        3   1                       173.958036    3.036141

3    3        0.500000         0.809017          0.309017
4    0        0.395511         0.343074          0.851981
4    1        0.499636         0.466634          0.729806

LENGTH          4    1        4   0                          0.20257315
AXIAL         0.0  0.0        4   0        4   1           84.186736    1.469336
FACE            4    1        3   0        4   0           61.609346    1.075286
FACE            4    1        5   1        4   0           59.236003    1.033863
DIHEDRAL        4    1        4   0                       173.371073    3.025896

4    2        0.577350         0.577350          O.577350

LENGTH          4    2        4   1                          0.20381459
AXIAL         0.0  0.0        4   1        4   2           84.150986    1.468712
FACE            4    2        3   1        4   1           61.316433    1.070174
FACE            4    2        5   2        4   1           59.027879    1.030231
DIHEDRAL        4    2        4   1                       173.262043    3.023993

4    3        0.624919         0.669347          0.401810
4    4        0.639950         0.738585          0.212031
5    0        0.468576         0.174499          0.866018
5    1        0.581020         0.291736          0.759807

LENGTH          5    1        5   0                          0.19408489
AXIAL         0.0  0.0        5   0        5   1           84.431107    1.473601
FACE            5    1        4   0        5   0           57.869432    1.010012
FACE            5    1        6   1        5   0           56.243358    0.981632
DIHEDRAL        5    1        5   0                       172.752892    3.015107

5    2        0.669347         0.401810          0.624919

LENGTH          5    2        5   1                          0.195222481
AXIAL         0.0  0.0        5   1        5   2           84.398302    1.473028
```

```
FACE            5   2       4   1       5   1            57.502420    1.003607
FACE            5   2       6   2       5   1            55.905183    0.975730
DIHEDRAL        5   2       5   1                       172.611080    3.012632

5   3        0.729806         0.499636         0.466634

LENGTH          5   3       5   2                         0.19565051
AXIAL        0.0  0.0       5   2       5   3            84.386042    1.472814
FACE            5   3       4   2       5   2            57.367121    1.001245
FACE            5   3       6   3       5   2            55.788106    0.973686
DIHEDRAL        5   3       5   2                       172.559191    3.011726

5   4        0.759807         0.581020         0.291736
5   5        0.758171         0.643075         0.107846
6   0        0.525731         0.000000         0.850651
6   1        0.643075         0.107846         0.758171

LENGTH          6   1       6   0                         0.18426310
AXIAL        0.0  0.0       6   0       6   1            84.713752    1.478534
FACE            6   1       5   0       6   0            54.176847    0.945564
DIHEDRAL        6   1       6   0                       172.340950    3.007917

6   2    0.738584 0.212031    0.639950

LENGTH          6   2       6   1                         0.18426310
AXIAL        0.0  0.0       6   1       6   2            84.713747    1.478534
FACE            6   2       5   1       6   1            53.512200    0.933964
DIHEDRAL        6   2       6   1                       172.151842    3.002795

6   3        0.809017         0.309017         0.500000

LENGTH          6   3       6   2                         0.18426311
AXIAL        0.0  0.0       6   2       6   3            84.713752    1.478534
FACE            6   3       5   2       6   2            53.150170    0.927645
DIHEDRAL        6   3       6   2                       172.047482    3.002795

6   4        0.851981         0.395511          0.343074
6   5        0.866018         0.468576          0.174499
6   6        0.850551         0.525731         -0.000000

ONE FACE
     AREA                     0.623048
     VOLUME                   0.206274
     LENGTH                  12.489358

COPPLETE STRUCTURE
     AREA                    12.460959
     VOLUME                   4.125473
     LENGTH                 216.619799
```

APPENDIX II

The plot routine was written for the IBM 7044 computer utilizing FORTRAN language. The program generates data necessary for input tapes to drive a CalComp plotter model 470. The example of the output map is for a six frequency Icosahedral sphere oriented in three directions as shown in Figures 15, 16, and 17.

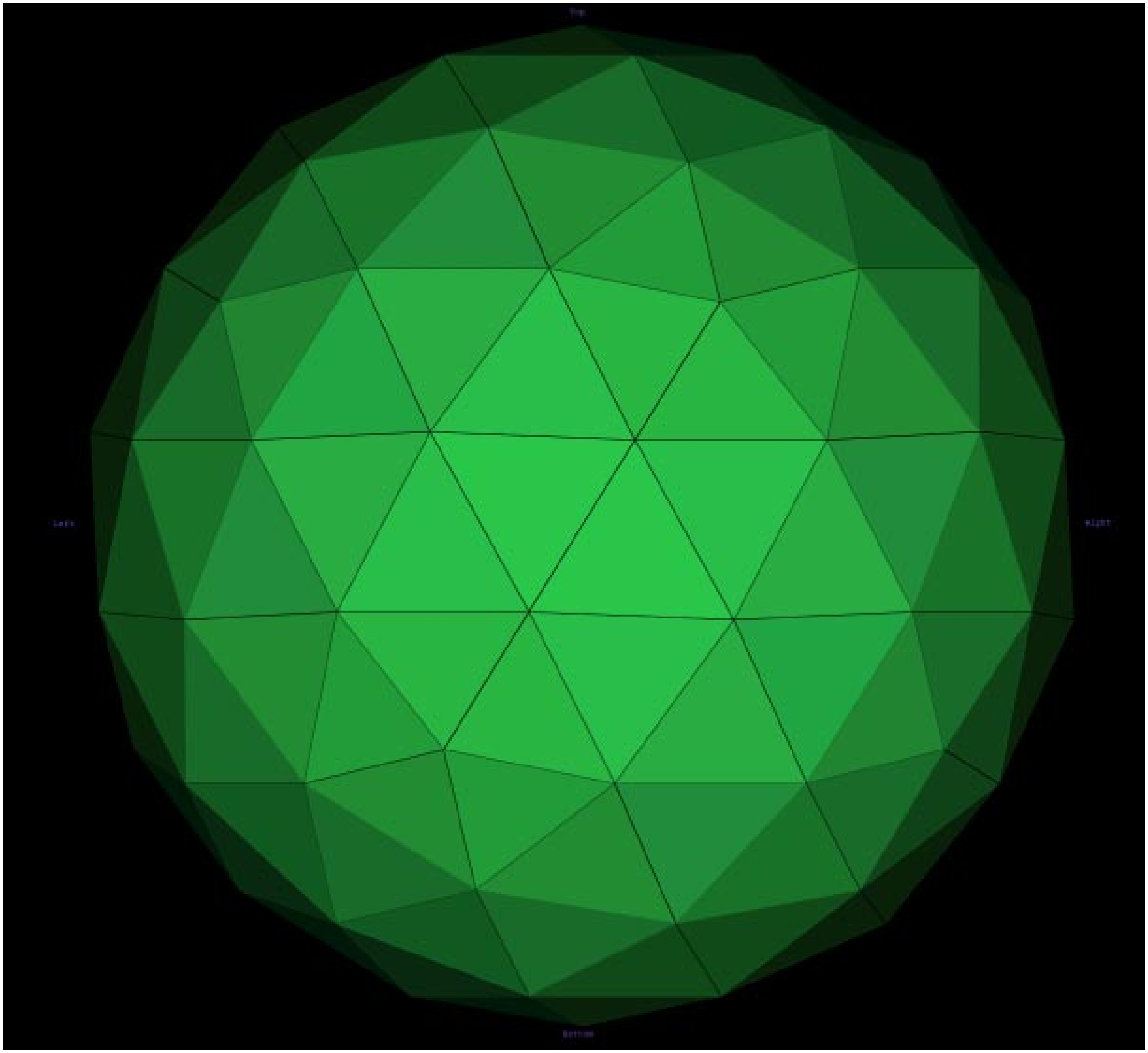
```
C        1         2         3         4         5         6         7         8
C     789012345678901234567890123456789012345678901234567890123456789012345678901234567890
$IBFTC PTTOI NODECK
      DIMENSION COORD(3, 3, 32), A(3, 3, 8), B(3, 3, 8), F(3, 3, 8),
     1 D(3, 3, 8), BUFF(4000), ALPHA(8), V(3, 3), LAB(2, 3), P(3, 3),
     2 SD 1(3, 32), SD 2(3, 32), SD 3(3, 32), EX(768, 3), EXP(768, 3),
     3 C(3, 3)
      COMMON DC(6, 3), X2R2, Y2R2, Z2R2, X2R1, Y2R1, X1R1
      EQUIVALENCE (COORD(1, 1, 1), A), (COORD(1, 1, 9), B),
     1 (COORD(1, 1, 17), F), (COORD(1, 1, 25), D),
     2 (V(1, 1), X1), (V(2, 1), Y1), (V(3, 1), Z1),
     3 (V(1, 2), X2), (V(2, 2), Y2), (V(3, 2), Z2),
     4 (V(1, 3), X3), (V(2, 3), Y3), (V(3, 3), Z3),
     5 (LAB(1, 1), I1), (LAB(2, 1), J1), (LAB(1, 2), I2),
     6 (LAB(2, 2), J2), (LAB(1, 3), I3), (LAB(2, 3), J3)
      LOGICAL OUT
      INTEGER RU, TU, RAD, TE OC IC, Q
      COEF(A1, A2, A3, B1, B2, B3) = A1 * (B2 - B3) - A2 * (B1 - B3) +
     1  A3 * (B1 - B2)
      DATA A /     -.57735027, -.57735027,  .57735027,  .57735027,
     1 -.57735027, -.57735027, -.57735027,  .57735027, -.57735027,
     2 -.57735027, -.57735027,  .57735027,  .57735027,  .57735027,
     3  .57735027,  .57735027, -.57735027, -.57735027, -.57735027,
     4 -.57735027,  .57735027, -.57735027,  .57735027, -.57735027,
     5  .57735027,  .57735027,  .57735027,  .57735027,  .57735027,
     6  .57735027, -.57735027,  .57735027, -.57735027,  .57735027,
     7 -.57735027, -.57735027,1.,0.,0.,0.,0.,1.,0.,1.,0.,1.,0.,0.,0.,
     8 1.,0.,0.,0.,-1.,1.,0.,0.,0.,0.,-1.,0.,-1.,0.,
     9 1.,0.,0.,0.,-1.,0.,0.,0.,1./
      DATA B / -1., 0.,0.,0.,0.,1.,0.,-1.,0.,
     1 -1.,0.,0.,0.,1.,0.,0.,0.,1.,
     2 -1.,0.,0.,0.,0.,-1.,0.,1.,0.,
     3 -1.,0.,0.,0.,-1.,0.,0.,0.,-1.,
     4 0.,.85065081,.52573111,.52573111,0.,.85065081,.85065081,
     5 .52573111,0.,0.,-,85065081,.52573111,.85065081,-.52573111,0.,
     6 .52573111,0.,.85055081,0.,-.8506508l,.52573111,-.52573111,0.,
     7 .85065081,-.85065081,-.52573111,0.,0.,.85065081,.52573111,
     8 -.85065081,.5257311l,0.,-.52573111,0.,.85065081 /
      DATA F / 0., .85065081,-.52573111, .85065081, .52573111, 0., .5257
     13111,0.,-.85065081,0.,-.85065081,-.52573111,.52573111,0.,-.8506508
     21,.85065081,-.52573111,0.,0.,-.85065081,-.52573111,-.85065081,-.52
     3573111,0.,-.52573111,0.,-.85065081,0.,.85065081,—.52573111,-.52573
     4111,0.,-.85065081,-.85065081,.52573111,0.,0.,.85065081,.52573111,-
     5.52573111,0.,.85065081,.52573111,0.,.85065081,0.,-.85065081,.52573
     6111,.52573111,0.,.85065081,-.52573111,0.,.85065081,0.,.85065081,-.
     752573111,.52573111,0.,-.85065081,-.52573111,0.,-.85065081,0.,-.850
     865081,-.52573111,-.52573111,0.,-.85065081,.52573111,0.,-.85065081/
     DATA D / O., .85065081, .52573111, .85065081, .52573111,0.,0.,.850
     165081,-.52573111,0.,.85065081,.52573111,0.,.85065081,-.52572111,-.
     285065081,.52573111,0.,0.,-.85065081,.52573111,0.,-.85065081,-.5257
     33111,.85065081,-.52573111,0.,0.,-.85065081,.52573111,-.85065081,-.
     452573111,0.,0.,-.85065081,-.52573111,.85065081,.52573111,0.,.52573
     5111,0.,.85065081,.85065081,-.52573111,0.,.85065081,.52573111,0.,.8
     65065081,-.52573111,0.,.52573111,0.,-.85065081,-.85065081,.52573111
     7,0.,-.85065081,-.52573111,0.,-.52573111,0.,.85065081,-.85065081,.5
     82573111,0.,-.52573111,0.,-.85065081,—.85065081,-.52573111,0. /
      DATA RAD / 1HR /
      CALL PLOTS (BUFF, 4000)
      CALL PLOT (0., 11.5, 23)
      CALL PLOT (0., -11., 21)
      CALL PLOT (6., -.5, 23)
    1 READ (5, 70) N, TE OC IC, ROT, RU, TILT, TU, ALPHA, METHOD
   70 FORMAT (212, 6X, 2(F9.0, A1), 8A6, I2)
      IF (N .EQ. 0) STOP
      CALL PLOT (0., 5-5, 23)
      CALL SYMBOL (-5., -4.50, .14, 10HFREQUENCY, 0., 10)
      CALL NUMBER (-0., -0., .14, FLOAT(N), 0., -1)
      CALL SYMBOL (-5., -4.75, .14, 9HROTATION , 0., 9)
      CALL NUMBER (-0., -0., .14, ROT, 0., 3)
      IF (RU .NE. RAD) GO TO 112
      CALL SYMBOL (-0., -0., .14, 8H RADIANS, 0., 8)
      GO TO ll3
  112 CALL SYMBOL (-0., -0., .14, 8H DEGREES, 0., 8)
  113 CALL SYMBOL (-5., -5.00, .14, 5HTILT , 0., 5)
      CALL NUMBER (-0., -0., .14, TILT, 0., 3)
      IF (TU .NE. RAD) GO TO 114
      CALL SYMBOL (-0., -0., .14, 8H RADIANS, 0., 8)
      GO TO 115
  114 CALL SYMBOL (-0., -0., .14, 8H DEGREES, 0., 8)
  115 CALL SYMBOL (-5., -5.25, .14, ALPHA, 0., 48)
      IF (RU .NE. RAD) ROT = ROT * .017453293
      IF (TU .NE. RAD) TILT = TILT * .017453293
      CALL DATE (4., -5., .14)
      CALL FACTOR (5.)
      CALL SYMBOL (-1.2, -1., .05, 3, 0., -1)
      FLOAT N = N
      SIN R = SIN(ROT)
      COS R = COS(ROT)
      SIN T = SIN(TILT)
      COS T = COS(TILT)
      MIN = 1
      HAX = 4
      IF (TE OC IC .NE. 8) GO TO 2
      MIN = 5
      MAX = 12
    2 IF (TE OC IC .NE. 20) GO TO 3
      MIN = 13
      MAX = 32
    3 DO 4 I = 1, 3
      DO 4 J = 1, 3
    4 V(J, I) = COORD(J, I, MIN)
      L1 = 0
      DO 7 L2 = 1, 2
      DO 7 LN = 2, 3
      IF (L2 .EQ. LN) GO TO 7
      L1 = L1 + 1
      CALL ROTATE (1, V(1, LN), V(2, LN), V(3, LN), V(1, L2), V(2, L2),
     * V(3, L2))
      THETA = ARCTAN2(Y2R1, X2R1)
      DO 7 L4 = 1, N
      T = L4
      ANG = T * THETA / FLOAT N
      X = COS(ANG)
      Y = SIN(ANC)
      A1 = -Y
      B1 = X
      A2 = —Y2R1
      B2 = X2R1 - 1.
      C2 = B2
      X = (B1*C2) / (A2*B1 - A1*B2)
      Y = (AI*C2) / (A1*H2 - A2*B1)
      CALL ROTATE (3, X, Y, 0.)
      IF(L1 - 2) 45, 5, 6
   45 SD 1(1, L4) = X2R2
      SD 1(2, L4) = Y2R2
      SD 1(3, L4) = Z2R2
      GO TO 7
    5 SD 2(1, L4) = X2R2
      SD 2(2, L4) = Y2R2
      SD 2(3, L4) = Z2R2
      GO TO 7
    6 SD 3(1, L4) = X2R2
      SD 3(2, L4) = Y2R2
      SD 3(3, L4) = Z2R2
    7 CONTINUE
      Q = 0
      DO 16 I1 = 1, N
      DO 16 Jl = 1, I1
      I2 = I1 - 1
      J2 = Jl - 1
      I3 = I1
      J3 = J2
      OUT = .FALSE.
    8 DO 13 LA = 1, 3
      LI = LAB(1, LA)
      LJ = LAB(2, LA)
      IJ = LI - LJ
      NJ = N - LJ
      IF (LI .EQ. LJ) GO TO 9
      IF (LJ .EQ. 0) GO TO 11
      IF (LI .NE. N) GO TO 12
      P(LA, 1) = SD 3(1, NJ)
      P(LA, 2) = SD 3(2, NJ)
      P(LA, 3) = SD 3(3, NJ)
      GO TO 13
    9 IF (LJ .EQ. 0) GO TO 10
      P(LA, 1) = SD 1(1, LI)
      P(LA, 2) = SD 1(2, LI)
      P(LA, 3) = SD 1(3, LI)
      GO TO 13
   10 P(LA, 1) = X1
      P(LA, 2) = Y1
      P(LA, 3) = Z1
      GO TO 13
   11 P(LA, 1) = SD 2(1, LI)
      P(LA, 2) = SD 2(2, LI)
      P(LA, 3) = SD 2(3, LI)
      GO TO 13
   12 CALL INTERC (SD 1(1, LI), SD 1(2, LI), SD 1(3, LI), SD2(1, LI),
     1 SD 2(2, LI), SD 2(3, LI), SD 3(1, NJ), SD 3(2, NJ), SD 3(3, NJ),
     2 SD 1(1, LJ), SD 1(2, LJ), SD 1(3, LJ), C(1, 1), C(1, 2), C(1,3))
      CALL INTERC (SD 3(1, IJ), SD 3(2, IJ), SD 3(3, IJ), SD 2(1, IJ),
     1 SD 2(2, IJ), SD 2(3, IJ), SD 3(1, NJ), SD 3(2, NJ), SD 3(3, NJ),
     2 SD 1(1, LJ), SD 1(2, LJ), SD 1(3, LJ), C(2, 1), C(2, 2), C(2,3))
      CALL INTERC (SD 3(1, IJ), SD 3(2, IJ), SD 3(3, IJ), SD 2(1, IJ),
     1 SD 2(2, IJ), SD 2(3, IJ), SD 1(1, LI), SD 1(2, LI), SD 1(3, LI),
     2 SD 2(1, L1), SD 2(2, LI), SD 2(3, LI), C(3, 1), C(3, 2), C(3,3))
      GO TO (1, 99, 100), METHOD
   99 P(LA, 1) = C(1, 1) + C(2, 1) + C(3, 1)
```

```
      P(LA, 2) = C(1, 2) + C(2, 2) + C(3, 2)
      P(LA, 3) = C(1, 3) + C(2, 3) + C(3, 3)
      GO TO 13
  100 DO 101 L = 1, 3
      DENOM = SQRT(C(L,1)**2+C(L,2)**2+C(L,3)**2)
      DO 101 M=1,3
  101 C(L,M)=C(L,M)/DENOM
C     VERTICES OF WINDOW HAVE BEEN EXPLODED TO SURFACE OF SPHERE.
      DO 102 L=1, 2
      DO 102 M=1, 3
  102 C(L,M)=C(L,M)-C(3,M)
C     WINDOW HAS BEEN TRANSLATED.8VERTEX C,I.E. VERTEX 3 IS AT 0,0,0.
      CALL ROTATE (1, C(1, 1), C(1, 2), C(1, 3), C(2, 1), C(2, 2),
     * C(2, 3))
      ANG = .5 * ATAN2(Y2R1,X2R1)
      XR1 = X2R1 * COS(ANG) + Y2R1 * SIN(ANG)
      YRI = Y2R1 * COS(ANG) - X2R1 * SIN(ANG)
      ANG = ATAN2(Y2R1,X1R1- X2R1)
      ANG = .5 * ANG
      XR2 = (X2R1- X1R1) * COS(ANG) - Y2R1 * SIN(ANG) + X1R1
      YR2 = Y2R1 * COS(ANG) + (X2R1- X1R1) * SIN(ANG)
      S1 = YR1 / XR1
      S2 = YR2 / (XR2 - X1R1)
      C2 = X1R1* S2
      D9 = - S1 + S2
      CX = C2 / D9
      CY = (S1 * C2) / D9
      CALL ROTATE (3, CX, CY, 0.0)
C     CENTER OF WINDOW HAS BEEN FOUND. TRANSLATE BACK. . .8
      P(LA, 1) = X2R2 + C(3, 1)
      P(LA, 2) = Y2R2 + C(3, 2)
      P(LA, 3) = Z2R2 + C(3, 3)
   l3 CONTINUE
      DO 14 IN1 = 1, 3
      R = SQRT(P(IN1, 1)**2 + P(IN1, 2)**2 + P(IN1, 3)**2)
      Q = Q + 1
      DO 14 IN2 = 1, 3
   14 EX (Q, IN2) = P(IN1, IN2) / R
      I3 = I2
      J2 = J2 + 1
      IF (OUT) GO TO 16
      IF (J2 .GT. 12) GO TO 16
      OUT = .TRUE.
      GO TO 8
   16 CONTINUE
      WRITE (6, 73) Q
   73 FORMAT (5H Q = I4)
      TX1 = X1 - X3
      TY1 = Y1 - Y3
      TZ1 = Z1 - Z3
      TX2 = X2 - X3
      TY2 = Y2 - Y3
      TZ2 = Z2 - Z3
      DO 17 K = 1, Q
      DO 17 K1 = 1, 3
   17 EXP(K, K1) = EX(K, K1) - V(K1, 3)
      CALL ROTATE (1, TX2, TY2, TZ2, TX1, TY1, TZ1)
      DO 23 M = MIN, MAX
      IF (M .EQ. MIN) GO TO 21
      DO 18 I = 1, 3
```

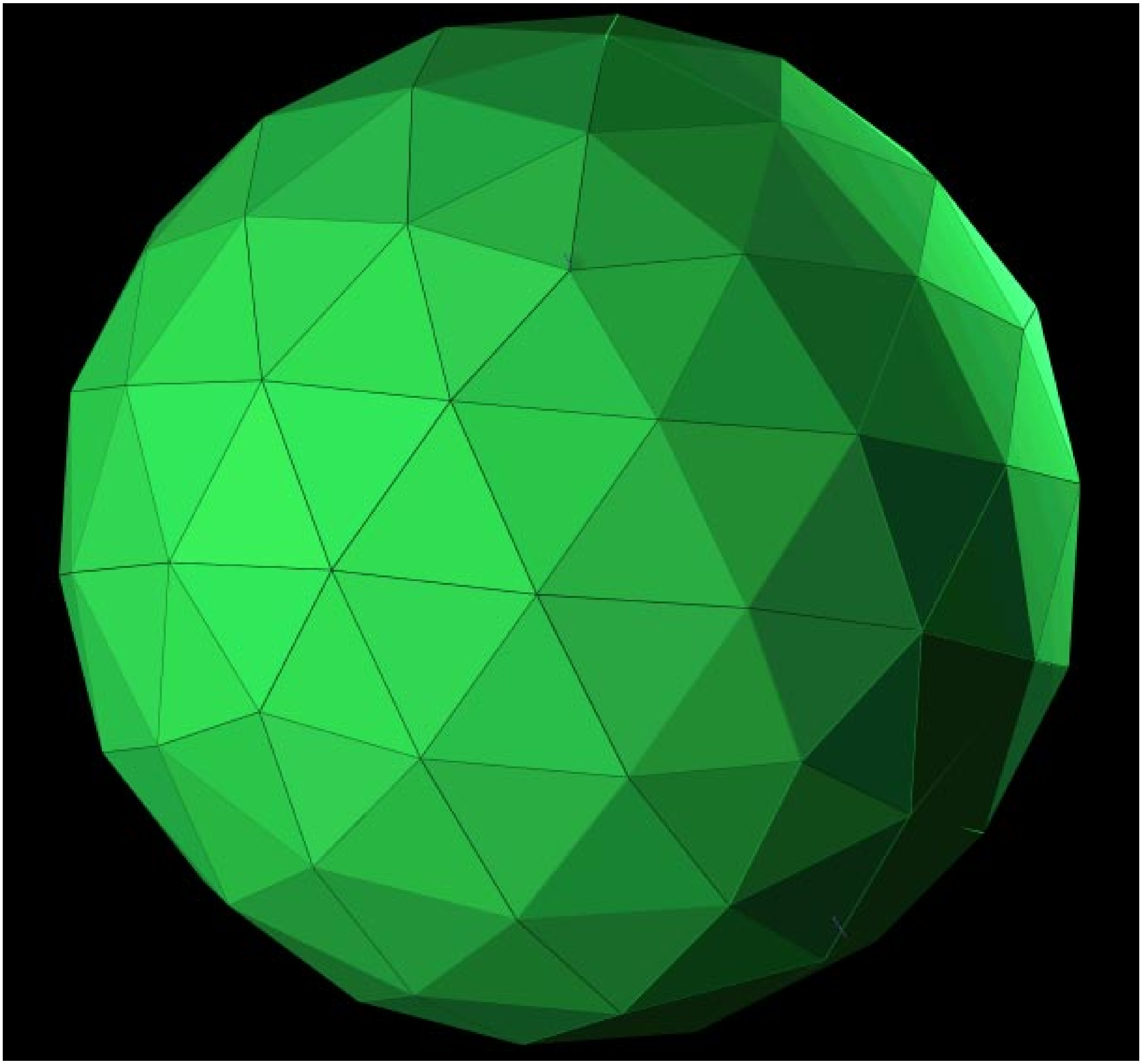
```
    DO 18 J = 1, 3
 18 V(J, I) = COORD(J, I, M)
    TX1 = X1 - X3
    TY1 = Y1 - Y3
    TZ1 = Z1 - Z3
    TX2 = X2 - X3
    TY2 = Y2 - Y3
    TZ2 = Z2 - Z3
    CALL ROTATE (2, TX2, TY2, TZ2, TX1, TY1, TZ1)
    DO 19 K = 1, 3
    DO 19 I = 1, 3
 19 P(I, K) = DC(1, K)*DC(4, 1) + DC(2, K)*DC(5, I) + DC(3, K)*DC(6,I)
    DO 20 K = 1, Q
    DO 20 L = 1, 3
 20 EX(K,L) = EXP(K,1)*P(L,1)+EXP(K,2)*P(L,2)+EXP(K,3)*P(L,3) * V(L,3)
 21 DO 22 K1 = 1, Q
    YRT = EX(K1,2) * COS R - EX(K1,1) * SIN R
    ZRT = EX(K1,3) * COS T - EX(K1,1) * COS R * SIN T - EX(K1, 2) *
   * SIN R * SIN T
    EX(K1, 2) = YRT
 22 EX(K1, 3) = ZRT
    DO 23 K1 = 1, Q, 3
    K2 = K1 + 1
    K3 = K1 + 2
    IF (COEF(EX(K1, 2), EX(K2, 2), EX(K3, 2), EX(K1, 3), EX(K2, 3),
   * EX(K3, 3)) .LE. 0.) GO TO 23
    CALL PLOT (EX(K1, 2), EX(K1, 3), 3)
    CALL PLOT (EX(K2, 2), EX(K2, 3), 2)
    CALL PLOT (EX(K3, 2), EX(K3, 3), 2)
    CALL PLOT (EX(K1, 2), EX(K1, 3), 2)
    WRITE (6, 705) (EX(NY, 2), EX(NY, 3), NY = K1, K3)
705 FORMAT (1X, 6F14.6)
 23 CONTINUE
    CALL SYMBOL (-1.2, -1., .03, 3, 0., -1)
    CALL FACTOR (1.)
    CALL PLOT (8.875, 11., 3)
    CALL PLOT (8.875, 0., 1)
    CALL PLOT (14.875, -.5, -3)
```

```
$IBFTC ROTATE NODECK
      SUBROUTINE ROTATE (N, X2, Y2, Z2, X1, Y1, Z1)
      COMMON DC(6, 3), X2R2, Y2R2, Z2R2, X2R1, Y2R1, X1R1
      IF (N .EQ. 3) GO TO 1
      M1 = 3*N - 2
      M2 = M1 + 1
      M3 = M2 + 1
      D1 = SQRT(X1**2 + Y1**2 + Z1**Z)
      DC(M1, 1) = X1 / D1
      DC(M1, 2) = Y1 / D1
      DC(M1, 3) = Z1 / D1
      H3 = Y1 * Z2 - Y2 * Zl
      U3 = Z1 * X2 - X1 * Z2
      V3 = X1 * Y2 - X2 * Y1
      D2 = SQRT(H3**2 + U3**2 + V3**2)
      DC(M3, 1) = H3 / D2
      DC(M3, 2) = U3 / D2
      DC(M3, 3) = V3 / D2
      H2 = DC(M3, 2) * Z1 - Y1 * DC(M3, 3)
      UZ = DC(H3, 3) * X1 - Z1 * DC(M3, 1)
      VZ = DC(H3, 1) * Y1 - X1 * DC(M3, 2)
      D3 = SQRT(H2**2 + U2**2 + V2**2)
      DC(M2, 1) = H2 / D3
      DC(M2, 2) = U2 / D3
      DC(M2, 3) = V2 / D3
      X1R1 = Xl*DC(M1, 1) + Y1*DC(M1, 2) + Z1*DC(M1, 3)
      X2R1 = X2*DC(M1, 1) + Y2*DC(M1, 2) + Z2*DC(M1, 3)
      Y2R1 = X2*DC(M2, 1) + Y2*DC(M2, 2) + Z2*DC(M2, 3)
      RETURN
    1 X2R2 = X2*DC(M1, 1) + Y2*DC(M2, 1) + Z2*DC(M3, 1)
      Y2R2 = X2*DC(M1, 2) + Y2*DC(M2, 2) + Z2*DC(M3, 2)
      Z2R2 = X2*DC(M1, 3) + Y2*DC(M2, 3) + Z2*DC(M3, 3)
      RETURN
      END
```

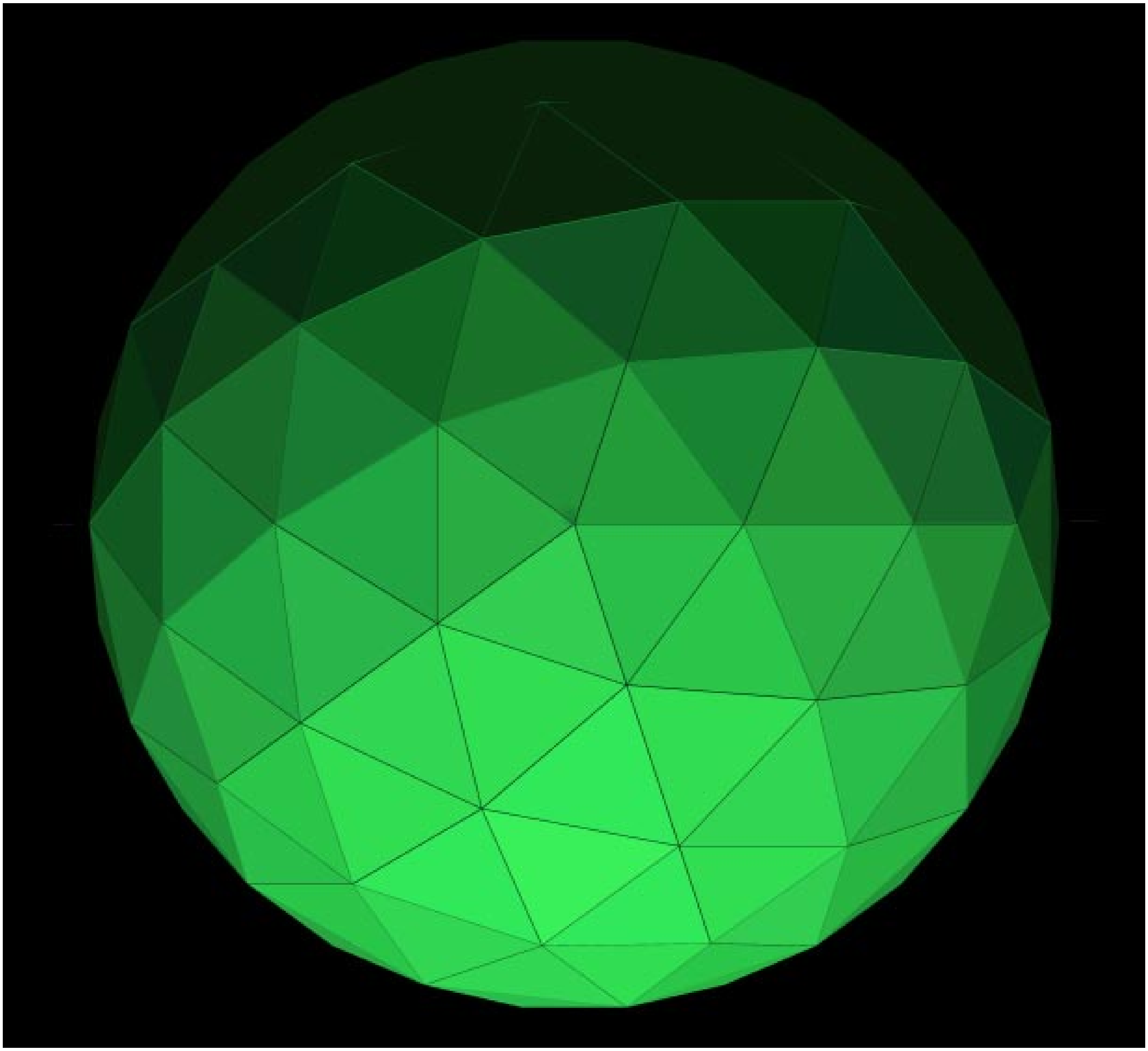
```
$IBFTC INTERC NODECK
      SUBROUTINE INTERC (X1, Y1, Z1, X2, Y2, Z2, X3, Y3, Z3, X4, Y4, Z4,
     * X, Y, Z)
      A1 = Y2 - Y1
      B1 = X1 - X2
      C1 = X1*(Y1 - Y2) + Y1*(X2 - X1)
      A2 = Y4 - Y3
      B2 = X3 - X4
      C2 = X3*(Y3 - Y4) + Y3*(X4 - X3)
      A3 = A1
      B3 = Z1 - Z2
      C3 = Z1*(Y1 - Y2) + Y1*(Z2 - Z1)
      A4 = A2
      B4 = Z3 - Z4
      C4 = Z3*(Y3 - Y4) + Y3*(Z4 - Z3)
      X = (B2*C1 - B1*C2) / (A2*B1 - A1*B2)
      Y = (A2*C1 - A1*C2) / (A1*B2 - A2*B1)
      Z = (B4*C3 - B3*C4) / (A4*B3 - A3*B4)
      RETURN
      END
```



3 FREQUENCY ICOSAHEDRON
METHOD II
EQUITORIAL VIEW



3 FREQUNCY ICOSAHEDRON
METHOD II
FACE VIEW



3 FREQUNCY ICOSAHEDRON
METHOD II
VERTICE VIEW

Author’s note:
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